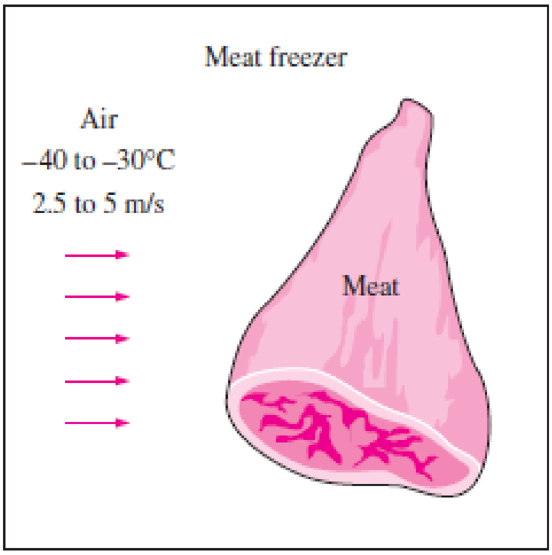
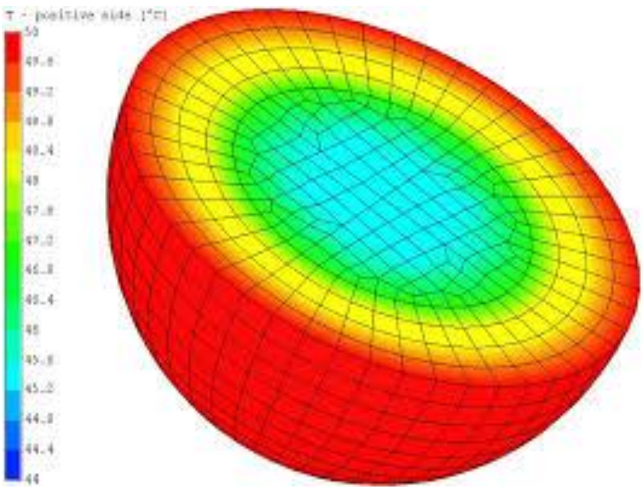
4

TRANSIENT HEAT CONDUCTION





Course Contents

4.1 Introduction

- In the preceding chapter, we considered heat conduction under *steady* conditions, for which the temperature of a body at any point does not change with time. This certainly simplified the analysis.
- But before steady-state conditions are reached, some time must elapse when a solid body is suddenly subjected to a change in environment. During this transient period the temperature changes, and the analysis must take into account changes in the internal energy.
- This study is a little more complicated due to the introduction of another variable namely time to the parameters affecting conduction. This means that temperature is not only a function of location, as in the steady state heat conduction, but also a function of time, i.e. $t = f(x, y, z, \tau)$.
- Transient heat flow is of great practical importance in industrial heating and cooling, some of the applications are given as follow
  - i Heating or cooling of metal billets;
  - ii Cooling of I.C. engine cylinder;
  - iii Cooling and freezing of food;
  - iv Brick burning and vulcanization of rubber;
  - v Starting and stopping of various heat exchanger unit in power plant.
- Change in temperature during unsteady state may follow a periodic or a non-periodic variation.
- **Periodic variation**
- The temperature changes in repeated cycles and the conditions get repeated after some fixed time interval. Some examples of periodic variation are given follow
  - i Variation of temperature of a building during a full day period of 24 hous
  - ii Temperature variation in surface of earth during a period of 24 hours
  - iii Heat processing of regenerators whose packings are alternately heated by flue gases and cooled by air
- **Non-periodic variation**
- The temperature changes as some non-linear function of time. This variation is neither according to any definite pattern nor is in repeated cycles. Examples are:
  - i Heating or cooling of an ingot in a furnace
  - ii Cooling of bars, blanks and metal billets in steel works

4.2 Transient Conduction in Solids with Infinite Thermal Conductivity $\mathrm{k} \rightarrow \infty$ (Lumped Parameter Analysis)

- Even though no materials in nature have an infinite thermal conductivity, many transient heat flow problems can be readily solved with acceptable accuracy by assuming that the internal conductive resistance of the system is so small that the temperature within the system is substantially uniform at any instant.

- This simplification is justified when the external thermal resistance (Convection resistance) between the surface of the system and the surrounding medium is so large compared to the internal thermal resistance (Conduction resistance) of the system that it controls the heat transfer process.
- Consider a small hot copper ball coming out of an oven (Figure 4–1). Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time due to large thermal conductivity.
- Thus the temperature of the ball remains uniform at all times.

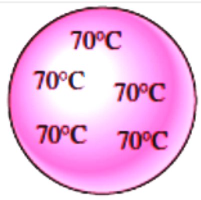


Fig. 4.1 Temperature distribution throughout the copper ball

- Consider a body of arbitrary shape of mass m, volume V, surface area A_s, density ρ, and specific heat C_p initially at a uniform temperature T_i (Figure 4–2).

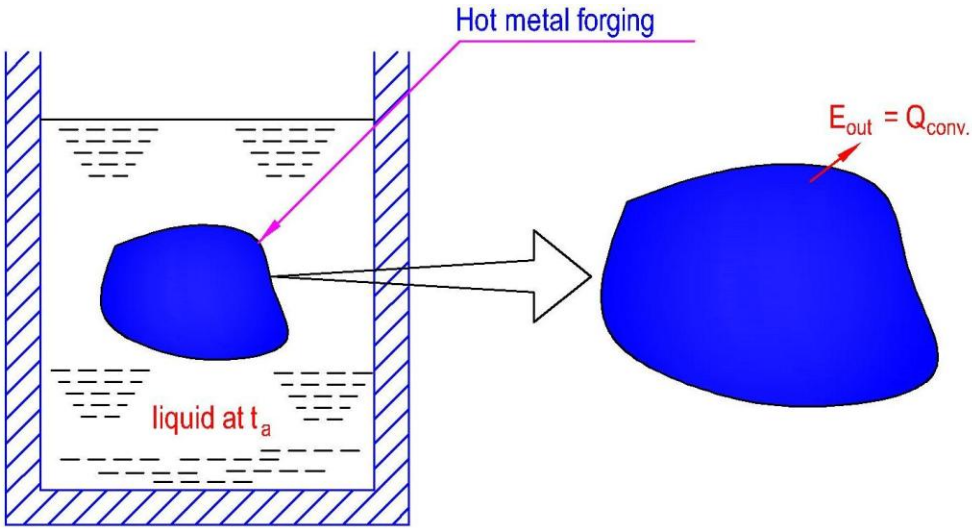


Fig. 4.2 Lumped parameter analysis

- At time τ = 0, the body is placed into a medium at temperature T_a, and heat transfer takes place between the body and its environment, with a heat transfer coefficient h. Let $T_i > T_a$, but the analysis is equally valid for the opposite case.
- During a differential time interval $d\tau$, the temperature of the body falls by a differential amount dT. An energy balance of the solid for the time interval $d\tau$ can be expressed as:

$$\begin{pmatrix} Heat\ transfer\ from\ body \\ by\ convection\ during\ d\tau \end{pmatrix} = \begin{pmatrix} The\ decrease\ in\ the\ energy\ of\ the\ body \\ during\ d\tau \end{pmatrix}$$

$$hA_s(T - T_a)d\tau = -mc\ dT$$

$$hA_s(T - T_a)d\tau = -\rho Vc\ dT$$

– Negative sign indicates the decrease in internal energy. This expression can be rearranged and integrated.

$$\int \frac{dT}{(T - T_a)} = -\frac{hA_s}{\rho Vc} \int d\tau$$

$$\ln(T - T_a) = -\frac{hA_s}{\rho Vc}\,\tau + C_1 - - - - - - - - - (4.1)$$

– The integration constant C_1 is evaluated from the initial conditions: $T = T_i \; at \; \tau = 0$. Substitute the value of boundary condition in equation 4.1, we get

$$C_1 = \ln(T_i - T_a)$$

– Substitute the value of C_1 in equation 4.1, we get

$$\ln(T - T_a) = -\frac{hA_s}{\rho Vc}\,\tau + \ln(T_i - T_a)$$

$$\ln\frac{(T - T_a)}{(T_i - T_a)} = -\frac{hA_s}{\rho Vc}\,\tau$$

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{hA_s}{\rho Vc}\tau\right) - - - - - - - - - (4.2)$$

– Equation 4.2 is used to find the temperature at any instant τ.
– Following points can be made from the above equations:

i The body temperature falls or rises exponentially with time and the rate depends on the parameter $(hA_s/\rho Vc)$. Theoretically the body takes infinite time to approach the temperature of surroundings and thus attain the steady state conditions. However the difference between T and T_a becomes extremely small after a short time and beyond that period the body temperature becomes practically equal to the ambient temperature. The change in temperature of a body with respect to time is shown in figure 4.3 for both cases (Heating and cooling)

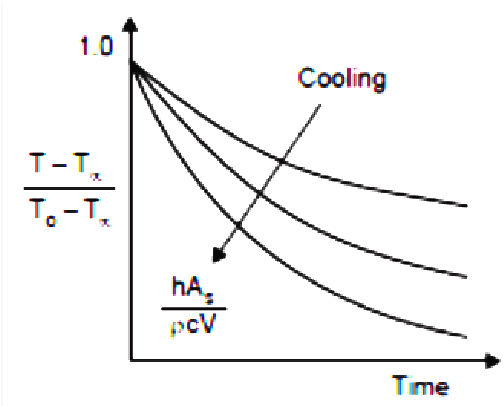


Fig. 4.3 Change in temperature of body with respect to time

ii The quantity $(\rho Vc/hA_s)$ has the dimensions of time and is called the thermal time constant. Its value is indicative of the rate of response of a system to a sudden change in the environmental temperature; how fast body will respond to a change in the environmental temperature. It should be as small as possible for fast response of the system to change in environmental temperature.

- Exponential term can be arranged in dimensionless term as follow:

$$\frac{hA_s}{\rho Vc}\tau = \left(\frac{hV}{kA_s}\right)\left(\frac{{A_s}^2 k}{\rho V^2 c}\tau\right)$$

$$= \left(\frac{hl}{k}\right)\left(\frac{\alpha\tau}{l^2}\right)$$

- Where, $\alpha = (k/\rho c)$ is the thermal diffusivity of the solid, and l is a characteristic length equal to the ratio of the volume of the solid to its surface area.
- The value of characteristic length of different geometry:

$$Sphere: l = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3}$$

$$Cylinder: l = \frac{\pi r^2 L}{2\pi r L} = \frac{r}{2}$$

$$Cube: l = \frac{L^3}{6L^2} = \frac{L}{6}$$

- The non-dimensional factor $(\alpha\tau/l^2)$ is called the Fourier number, F_o. It signifies the degree of penetration of heating or cooling effect through a solid. For instance, a large time τ would be required to obtain a significant temperature change for small values of $(\alpha\tau/l^2)$.
- The non-dimensional factor (hl/k) is called the Biot number, B_i. It gives the indication of the ratio of internal (conduction) resistance to the surface (convection) resistance.
- A small value of B_i implies that the system has a small conduction resistance, i.e. relatively small temperature gradient or nearly uniform temperature within the system. In that case heat transfer is predominates by convective heat transfer coefficient.
- **Criteria for Lumped System Analysis**
- Biot number is used to check the applicability of lumped parameter analysis. If Biot number is less than 0.1, it has been proved that this model can be used without appreciable error.
- The lumped parameter solution for transient conduction can be conveniently stated as

$$\frac{(T - T_a)}{(T_i - T_a)} = exp(-\,B_i F_o) --------(4.3)$$

- **Instantaneous and total heat flow rate**

- The instantaneous heat flow rate Q_i may be obtained by using Newton's law of cooling. Heat transfer from the body at any instant τ is given as:

$$Q_i = hA_s(T - T_a) - - - - - - - - -(4.4)$$

- Where T is the temperature at any instant τ. Substitute the value of $(T - T_a)$ from the equation no. 4.2. We get

$$Q_i = hA_s(T_i - T_a)\, exp\left(-\frac{hA_s}{\rho Vc}\tau\right) - - - - - - - - -(4.5)$$

- **Total heat flow rate**
- Total heat flow rate Q_t can be obtained by integrating the equation 4.5 over the time interval $\tau = 0\ to\ \tau = \tau$.

$$Q_t = \int_0^\tau Q_i\, d\tau$$

$$= \int_0^\tau hA_s(T_i - T_a)\, exp\left(-\frac{hA_s}{\rho Vc}\tau\right)\, d\tau$$

$$= \left[hA_s(T_i - T_a)\frac{exp[-(hA_s/\rho Vc)\,\tau]}{-\,hA_s/\rho Vc}\right]_0^\tau$$

$$= -\rho Vc(T_i - T_a)\left[exp\left(-\frac{hA_s}{\rho Vc}\tau\right)\right]_0^\tau$$

$$= -\rho Vc(T_i - T_a)\left[exp\left(-\frac{hA_s}{\rho Vc}\tau\right) - 1\right] - - - - - - - - -(4.6)$$

4.3 Time Constant and Response of a Thermocouple

- A Thermocouple is a sensor used to measure temperature. A thermocouple is comprised of at least two metals joined together to form two junctions.
- One is connected to the body whose temperature is to be measured; this is the hot or measuring junction. The other junction is connected to a body of known temperature; this is the cold or reference junction.
- Therefore the thermocouple measures unknown temperature of the body with reference to the known temperature of the other body.
- Measurement of temperature by a thermocouple is an important application of the lumped parameter analysis.
- The response of a thermocouple is defined as the time required for the thermocouple to reach the source temperature when it is exposed to it.
- Referring to the lumped-parameter solution for transient heat conduction;

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{hA_s}{\rho Vc}\tau\right) - - - - - - - - -(4.7)$$

- It is evident that larger the parameter $hA_s/\rho Vc$, the faster the exponential term will reach zero or more rapid will be the response of the thermocouple. A large value of $hA_s/\rho Vc$ can be obtained either by increasing the value of convective coefficient, or by decreasing the wire diameter, density and specific heat.
- The sensitivity of the thermocouple is defined as the time required by the thermocouple to reach 63.2% of its steady state value. According to definition of sensitivity

$$\frac{T - T_a}{T_i - T_a} = 1 - 0.632 = 0.368$$

- Substitute the value in equation 4.7

$$0.368 = exp\left(-\frac{hA_s}{\rho Vc}\tau\right)$$

$$\therefore \ln 0.368 = -\frac{hA_s}{\rho Vc}\tau$$

$$\therefore -\frac{hA_s}{\rho Vc}\tau = -1$$

$$\tau = \frac{\rho Vc}{hA_s}$$

- The parameter $\rho Vc/hA_s$ has units of time and is called time constant of the system and is denoted by τ^*. Thus

$$\tau^* = \frac{\rho Vc}{hA_s} - - - - - - - - - (4.8)$$

- Using time constant, the temperature distribution in the solids can be expressed as

$$\frac{\theta}{\theta_i}\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{\tau}{\tau^*}\right) - - - - - - - - - (4.9)$$

- The time constant represents the speed of response, i.e., how fast the thermocouple tends to reach the steady state value. A large time constant corresponds to a slow system response, and a small time constant represent a fast response. A low value of time constant can be achieved for a thermocouple by
  - i Decreasing light metals the wire diameter
  - ii Using light metals of low density and low specific heat
  - iii Increasing the heat transfer coefficient
- Depending upon the type of fluid used, the response times for different sizes and materials of thermocouple wires usually lie between 0.04 to 2.5 seconds.
- Note:- Once the time constant is measured, we have to wait for the that time to measure the temperature within 63.2% of accuracy.

4.4 Transient Heat Conduction In Solids With Finite Conduction and Convective Resistance ($0 < B_i < 100$)

- In the lumped parameter analysis we assume that conductivity of the material is infinite or variation of temperature within the body is negligible.
- But sometimes there may be variation of temperature with time and position.
- Consider a plane wall of thickness 2L, a long cylinder of radius r_o, and a sphere of radius r_o initially at a uniform temperature T_i, as shown in figure 4.4.
- Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetry about its center plane (x = 0), the cylinder is symmetry about its centerline (r = 0), and the sphere is symmetry about its center point (r = 0).

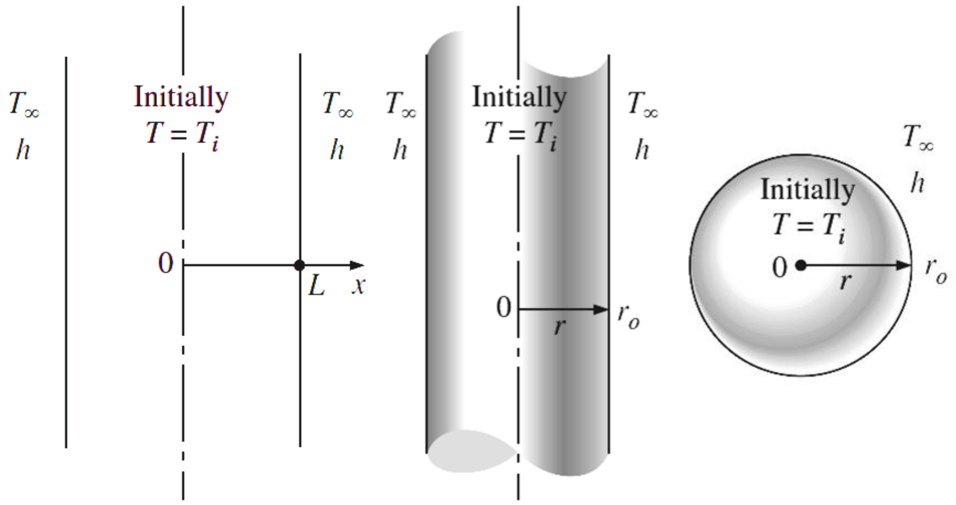


Fig. 4.4 Transient heat conduction in large wall, cylinder and sphere

- At a time $\tau = 0$, each geometry is placed in a large medium that is at a constant temperature T_∞. Heat transfer takes place between these bodies and their environments by convection with a uniform and constant heat transfer coefficient h.
- **Temperature profile of plane wall**
- The variation of temperature profile with respect to time in plane wall is shown in figure 4.5.

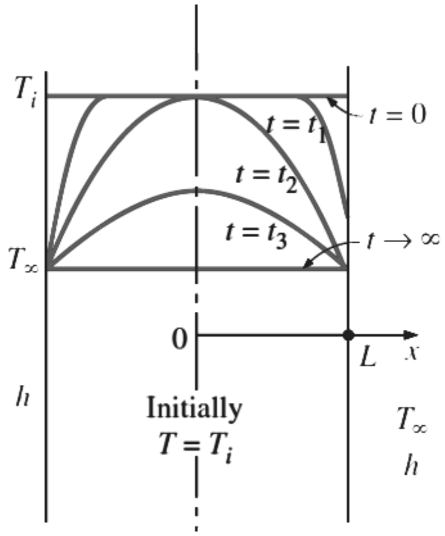


Fig. 4.5 Transient heat conduction in large wall, cylinder and sphere

- When the wall is first exposed to the surrounding medium the entire wall is at its initial temperature T_i.
- But the wall temperature at the surface starts to drop as a result of heat transfer from the wall to the surrounding medium. This creates a temperature gradient in the wall.
- The temperature profile within the wall remains symmetric at all times about the centre plane. The temperature profile gets flatter and flatter as times passes as a result of heat transfer and finally becomes uniform at $\tau = \tau_\infty$.
- The controlling differential equation for the transient heat conduction is:

$$\frac{d^2t}{dx^2} = \frac{1}{\alpha}\frac{dt}{d\tau}$$

- The appropriate boundary conditions are :
- $t = t_i$ at $\tau = 0$; initially the wall is at uniform temperature t_i
- $dt/dx = 0$ at $x = 0$; symmetrical nature of the temperature profile within the plane wall;
- $k\,A(dt/dx) = h\,A(t - t_a)$ at $x = \pm L$. At the surface heat transfer by conduction is equal to heat transfer by convection from the surface to medium.
- The solution of the controlling differential equation in conjunction with initial boundary conditions would give an expression for temperature variation both with time and position.
- The solution obtained after mathematical analysis indicate that

$$\frac{t - t_\infty}{t_i - t_\infty} = f\left(\frac{x}{l}, \frac{hl}{k}, \frac{\alpha\tau}{l^2}\right) - - - - - - - - -(4.10)$$

- The temperature history becomes a function of Biot number hl/k, Fourier number $\alpha\tau/l^2$ and the dimensionless parameter x/l which indicates the location of point within the plate where temperature is to be obtained. In case of cylinders and spheres x/l is replaced by r/R.

– The Heisler charts give the temperature history of the solid at its mid plane, $x = 0$. The temperatures at other locations are worked out by multiplying the mid-plane temperature by correction factors read from correction charts.
– Following relation is used to measure temperature at any location

$$\frac{t - t_\infty}{t_i - t_\infty} = \left(\frac{t_o - t_\infty}{t_i - t_\infty}\right) \cdot \left(\frac{t - t_\infty}{t_o - t_\infty}\right) - - - - - - - - - (4.11)$$

– The Heisler charts are extensively used to determine the temperature distribution and heat flow rate when both conduction and convection resistances are almost of equal importance.

4.5 Solved Numerical

Ex. 4.1.

A spherical element of 40 mm diameter is initially at temperature of 27°C. It is placed in boiling water for 4 minutes. After 4 minuts, at what temperature, the spherical element will reach? If the same spherical element is initially at 0°C, find out by lump theory that how much time will be taken by the element to reach at that temperature? Take properties of the given spherical element as:
$k = 10\, W/m$ °C, $\rho = 1200\, kg/m^3$, $C = 2\, kJ/kg$ °C and heat transfer coefficient $h = 100\, W/m^2$°C.

Solution:

Given data:

$d = 40\, mm = 40 \times 10^{-3}\, m$, $T_i = 27$°C , $T_a = 100$°C , $\tau = 4$ min. $= 240\, sec.$

$$\frac{A_s}{V} = \frac{4\pi r^2}{{}^4/_3\, \pi r^3} = \frac{3}{r} = \frac{6}{d}$$

a. Find the temperature of spherical element after 4 min.

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{hA_s}{\rho V c}\tau\right)$$

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{h}{\rho c}\left(\frac{A_s}{V}\right)\tau\right) = exp\left(-\frac{h}{\rho c}\left(\frac{6}{d}\right)\tau\right)$$

$$\frac{(T - 100)}{(27 - 100)} = exp\left(-\frac{100}{1200 \times 2000} \times \left(\frac{6}{40 \times 10^{-3}}\right) \times 240\right)$$

$$\frac{(T - 100)}{-73} = exp(-1.5)$$

$$T - 100 = 0.223 \times (-73) = -16.28$$

$$T = 100 - 16.28 = 83.71\text{°C}$$

b. Find the time required to reach desired temperature of 83.71°C when initial temperature is 0°C

$$\frac{(T-T_a)}{(T_i-T_a)} = exp\left(-\frac{hA_s}{\rho Vc}\tau\right)$$

$$\frac{(T-T_a)}{(T_i-T_a)} = exp\left(-\frac{h}{\rho c}\left(\frac{A_s}{V}\right)\tau\right) = exp\left(-\frac{h}{\rho c}\left(\frac{6}{d}\right)\tau\right)$$

$$\frac{(83.71-100)}{(0-100)} = exp\left(-\frac{100}{1200\times 2000}\times\left(\frac{6}{40\times 10^{-3}}\right)\times\tau\right)$$

$$\frac{-16.29}{-100} = exp(-6.25\times 10^{-3}\times\tau)$$

$$(-6.25\times 10^{-3}\times\tau) = \ln\left(\frac{16.29}{100}\right) = -1.815$$

$$\tau = \frac{-1.815}{-6.25\times 10^{-3}} = 290.4\ sec. = 4\min 50.4\ sec$$

Ex. 4.2.

During a heat treatment process, spherical balls of 12 mm diameter are initially heated to *800*°C. Then they are cooled to *100*°C by immersing them in an oil bath of *35*°C with convection coefficient $20\ W/m^2\ K$. Determine time required for cooling process. What should be the convection coefficient if it is intended to complete the cooling process in 10 minutes?

Thermo-physical properties of the balls are $\rho = 7750\,kg/m^3$, $C_p = 520\,J/kg\ K$, $k = 50\,W/m\ K$.

Solution:

Given data:

$d = 12\ mm = 12\times 10^{-3}\ m$, $T_i = 800$°C, $T_a = 35$°C, $T = 100$°C, $h = 20\ W/m^2\ K$

$$\frac{A_s}{V} = \frac{4\pi r^2}{{}^4/_3\,\pi r^3} = \frac{3}{r} = \frac{6}{d}$$

a. Find the time required to obtain the required temperature.

$$\frac{(T-T_a)}{(T_i-T_a)} = exp\left(-\frac{hA_s}{\rho Vc}\tau\right)$$

$$\frac{(T-T_a)}{(T_i-T_a)} = exp\left(-\frac{h}{\rho c}\left(\frac{A_s}{V}\right)\tau\right) = exp\left(-\frac{h}{\rho c}\left(\frac{6}{d}\right)\tau\right)$$

$$\frac{(100-35)}{(800-35)} = exp\left(-\frac{20}{7750\times 520}\times\left(\frac{6}{12\times 10^{-3}}\right)\tau\right)$$

$$\frac{65}{765} = exp(-2.48\times 10^{-3}\times\tau)$$

$$(-2.48 \times 10^{-3} \times \tau) = \ln\left(\frac{65}{765}\right) = -2.465$$

$$\tau = \frac{-2.465}{-2.48 \times 10^{-3}} = 993.95\ sec. = 16 \min 33.95\ sec$$

b. Find the convection co-efficient to complete the above process in 10 minutes.

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{hA_s}{\rho V c}\tau\right)$$

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{h}{\rho c}\left(\frac{A_s}{V}\right)\tau\right) = exp\left(-\frac{h}{\rho c}\left(\frac{6}{d}\right)\tau\right)$$

$$\frac{(100 - 35)}{(800 - 35)} = exp\left(-\frac{h}{7750 \times 520} \times \left(\frac{6}{12 \times 10^{-3}}\right) \times 600\right)$$

$$\frac{65}{765} = exp(-0.0744 \times h)$$

$$(-0.0744 \times h) = ln\left(\frac{65}{765}\right) = -2.465$$

$$h = \frac{2.465}{0.0744} = 33.13\ W/m^2\ K$$

Ex. 4.3.

The temperature of an air stream flowing with a velocity of 3 m/s is measured by a copper-constantan thermocouple which may be approximated as sphere of 3 mm in diameter. Initially the junction and air are at a temperature of *25*°C. The air temperature suddenly changes to and is maintained at *200*°C.

Take $\rho = 8685\ kg/m^3$, $C_p = 383\ J/kg\ K$, and $k = 29\ W/m\ K$ and $h = 150\ W/m^2\ K$. Determine: (i) Thermal time constant and temperature indicated by the thermocouple at that instant (ii) Time required for the thermocouple to indicate a temperature of *199*°C (iii) Discuss the suitability of this thermocouple to measure unsteady state temperature of fluid then the temperature variation in the fluid has a time period of 30 seconds.

Solution:

Given data:

$d = 3\ mm = 3 \times 10^{-3}\ m$, $T_i = 25$, $T_a = 200$°C, $h = 150\ W/m^2\ K$, $V = 3\ m/s$

$$\frac{A_s}{V} = \frac{4\pi r^2}{{}^4/_3\,\pi r^3} = \frac{3}{r} = \frac{6}{d}$$

i. Thermal time constant and temperature indicated by it at that instant

$$\tau^* = \frac{\rho V c}{h A_s}$$

$$\tau^* = \frac{\rho c}{h}\left(\frac{V}{A_s}\right) = \frac{\rho c}{h}\left(\frac{d}{6}\right)$$

$$\tau^* = \frac{8685 \times 383}{150}\left(\frac{3 \times 10^{-3}}{6}\right) = 11.09\ sec$$

Temperature at time $11.09\ sec$.

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{\tau}{\tau^*}\right)$$

$$\frac{(T - 200)}{(25 - 200)} = exp(-1)$$

$$\frac{(T - 200)}{-175} = 0.3678$$

$$T - 200 = 0.3678 \times (-175) = -64.365$$

$$T = 200 - 64.365 = 135.64℃$$

ii. Time required for the thermocouple to indicate the temperature of 199℃

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{\tau}{\tau^*}\right)$$

$$\frac{(199 - 200)}{(25 - 200)} = exp\left(-\frac{\tau}{11.09}\right)$$

$$\frac{(-1)}{(-175)} = exp\left(-\frac{\tau}{11.09}\right)$$

$$-\frac{\tau}{11.09} = \ln\left(\frac{1}{175}\right) = -5.165$$

$$\tau = 5.165 \times 11.09 = 57.277\ sec$$

Since, the time constant $(11.09\ sec)$ is less than the time for the temperature change of the fluid $(30\ sec)$, the thermometer will give a faithful record of the time varying temperature of the fluid.

4.6 References:

[1] Heat and Mass Transfer by D. S. Kumar, S K Kataria and Sons Publications.

[2] Heat Transfer – A Practical Approach by Yunus Cengel & Boles, McGraw-Hill Publication.