5

Inventory Control

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5.1 Introduction

In our daily life, we observe that a small retailer knows roughly the demand of his customer in a month or a week and accordingly places an order on the wholesaler to meet the demand of his customers. But this is not the case with a manager of a big departmental store or a big retailer because the stocking in such cases depends upon various factors e.g, demand, time of order, the time between orders and actual receipts, etc. So the real problem is to have a compromise between *over-stocking* and *under-stocking.*

Inventory management or Inventory Control is one of the techniques of Materials Management which helps the management to improve the productivity of capital by reducing the material costs, preventing the large amounts of capital from being locked up for long periods, and improving the capital-turn over ratio. The techniques of inventory control were evolved and developed during and after the *Second World War* and have helped the more industrially developed countries to make spectacular progress in improving their productivity.

Inventory: The word inventory means a physical stock of material or goods or commodities or other economic resources that are stored or reserved or kept in stock or in hand for smooth and efficient running of future affairs of an organization at the minimum cost of funds or capital blocked in the form of materials or goods (Inventories).

The function of directing the movement of goods through the entire manufacturing cycle from the requisitioning of raw materials to the inventory of finished goods in an orderly manner to meet the objectives of maximum customer service with minimum investment and efficient (low cost) plant operation is termed as ***Inventory Control.***

5.2 Classification of Inventories

Inventories may be classified as those which play a direct role during manufacture or which can be identified on the product and the second one are those which are required for manufacturing but not as a part of production or cannot be identified on the product. The first type is labeled as ***direct inventories*** and the second is labeled as ***indirect inventories.***

5.2.1 Direct Inventories

They include items that are directly used for production and are classified as:

a) **Production Inventory:** Items such as raw materials, components, and subassemblies used to produce the final product.
b) **Work in Process Inventory:** Items in semi-finished form or products at different stages of production.
c) **Finished Goods Inventory:** This includes the final products ready for dispatch to consumers or distributors.
d) **MRO Inventory:** Maintenance, repair and operating items such as spare parts and consumable stores that do not go into the final product but are consumed during the production process.
e) **Miscellaneous Inventory:** All other items such as scrap, obsolete and unsaleable products, stationery and other items used in office, factory and sales department, etc.

5.2.2 Indirect Inventories

They may be classified as:

a) **Transit or Pipeline Inventory or Movement Inventory:** They consist of items that are currently under transportation e.g, coal being transported from coalfields to the thermal power plant.

b) **Buffer Inventories:** These inventories are held to prevent stock-out due to uncertain demand and supply fluctuations. A company is generally aware of the average demand for various items it needs. However, the actual demand may exceed this average demand. Hence, inventories may be held in excess of the expected demand. Similarly, the average delivery time of an inventory is known to the company. However, unforeseen events may cause the actual delivery time to be more than the expected time. Thus, excess stock is held to meet fluctuations in demand and lead time.

c) **Decoupling Inventories:** They are required to decouple or disengage the different parts of the production system. For an item that requires processing on a series of different machines with different processing times, it is a must to have decoupling inventories of the item in between the various machines for smooth and continuous production. The decoupling inventories act as shock absorbers in case of varying work-rates, machine breakdowns or failures, etc.

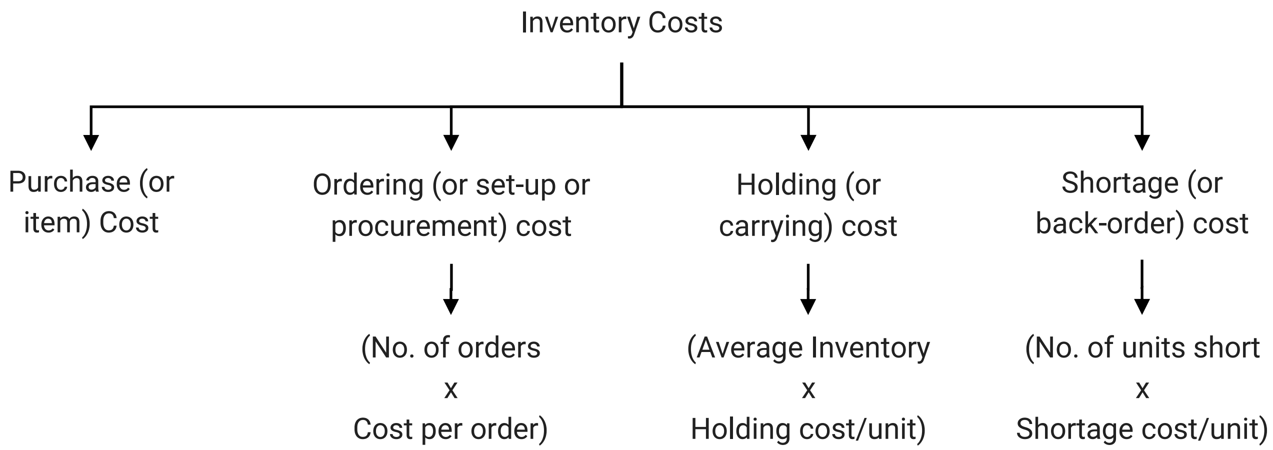
d) **Seasonal Inventories:** Some items have seasonal demand e.g. demand of woolen textile in winter, coolers and air conditioners in summer, raincoats in rainy season, etc. Inventories of such items have to be maintained to meet their high seasonal demand.

e) **Lot size or cycle inventories:** These inventories are held due to the fact that orders are placed in lots rather than purchasing the exact amount of inventory which may be needed at a point of time. If all purchases are made on as needed basis there would be no cyclic inventories but practically it may be more economical to carry amount of inventory by ordering or producing in large lots to achieve reduced ordering or set up costs to obtain quantity discounts on the items purchased.

f) **Anticipation Inventories:** These are built up in advance for a big selling season, a promotion program or a plant shut-down period. E.g., Purchasing of crackers well before Diwali, fans before the approaching summer, pilling up raw materials in the face of forthcoming transporters' strikes.

5.3 Different Costs associated with Inventories

While maintaining the inventories, we will come across certain costs associated with inventory. There are four major elements of inventory costs that should be taken for analysis.



5.3.1 Purchase Cost (P)

It is the actual price paid for purchasing/producing an item. The component of this cost includes direct material cost, direct labour cost, direct expenses, the profit of manufacturer.

The unit price P of an item is independent of the size of the quantity purchased/manufactured.

Purchase Cost = (Price per unit) × (Demand per unit time)

5.3.2 Ordering cost or Set-up cost or Procurement cost (C_o)

Administrative and clerical costs are involved in processing a purchase order, expediting, follow up, etc., It includes transportation costs also. When a unit is manufactured, the unit set up cost includes the cost of labor and materials used in the setup and set up testing and training costs. This is denoted by Rs. C_o per set up or per order.

5.3.3 Inventory carrying or Holding cost (C_h)

Holding cost (also known as carrying cost) represents the cost that is associated with storing an item in inventory. It is proportional to the amount of inventory and the time over which is held. It is usually expressed as a rate per unit or as a percentage of the inventory value. The components comprising the holding costs are explained below:

1. The cost of money or capital tied up in the Inventory, e.g, interest on the locked-up capital.
2. Storage cost includes cost of space, cost of building warehouses, cost of racks, fixtures, material handling equipment and salary of warehouse employees, cost of lighting the warehouse and providing the special environmental condition etc.
3. Damage or deterioration cost. E.g, cost of damage in storage and quality losses.
4. Insurance cost i.e., insurance premium to be paid while in storage.
5. Pilferage cost: Valuables like brasswares, gunmetal bushes, bearings, and oil seals, etc. are likely to be stolen (pilfer) from the store. One percent of inventory value may be assigned to pilferage cost.
6. Obsolescence cost: Depending on the nature of the item in stocks electronic goods and computer components are likely to be fast outdated; changes in the design also lead to obsolescence. Obsolescence cost may be taken for 4% of the stock value.
7. Handling costs include all costs associated with the movement of stock such as the cost of labour, overhead cranes, and other machinery used for this purpose.
8. Cost of maintaining inventory records.

5.3.4 Shortage Cost (C_S)

It is an extremely important cost that never appears in the accounting record. Some times it so happens that the material may not be available when needed or when the demand arises. In such cases the production has to be stopped until the procurement of the material, which may lead to missing the delivery dates or delayed production.

When the organization could not meet the delivery promises, it has to pay a penalty to the customer. If the situation of stock out will occur very often, then the customer may not come to the organization to place orders, that is the organization is losing the customers. In other words, the organization is losing the goodwill of the customers. The cost of goodwill cannot be estimated. In some cases it will be very heavy to such extent that the organization has to forego its business.

Here to avoid the shortage (or stock out) situation, if the organization stocks more material, inventory carrying cost increases and to take care of inventory cost, if the organization purchase just sufficient or less quantity, then the shortage position may arise. Hence the inventory manager must have sound knowledge of various factors that are related to inventory carrying cost and shortage cost and estimate the quantity of material to be purchased or else he must have effective strategies to face crucial situations. The cost is generally represented as so many rupees and is represented by (C_S).

5.4 Important Factors to be considered in Inventory Control

There are many factors, which have influence on the inventory, which draws the attention of an inventory manager, they are:

i) Demand

The demand for raw material or components for production or demand of goods to satisfy the needs of the customer, can be assessed from the past consumption/supply pattern of material or goods. We find that the demand may be deterministic in nature *i.e.*, we can specify that the demand for the item is so many units, for example, say '*q*' units per unit of time. Some times we find that the demand for the item may be probabilistic in nature *i.e.* we have to express in terms of expected quantity of material required for the period. Also the demand may be static, *i.e.* it means constant for each time period (uniform over equal period of times). Further, the demand may follow several patterns and so why it is uncontrolled variable, such as it may be uniformly distributed over period or instantaneous at the beginning of the period or it may be large at the beginning and less in the end, etc. These patterns directly affect the total carrying cost of inventory.

ii) Order Cycle

The time period between the placement of two successive orders is referred to as an order cycle.the order may be placed on the basis of following two types of inventory review system:

Continuous review: In this case, the record of the inventory level is updated continuously until a specified point (known as re-order point) is reached, at this point a new order is placed.

Periodic review: In this case, the orders are placed at equally spaced intervals of time. The quantity ordered each time depends on the available inventory level at the time of review.

iii) Time Horizon

The time period for which the optimal policy is to be formulated or the inventory cost is to be optimized is generally termed as the Inventory planning period or Time horizon.

iv) Lead Time or Delivery Lag

The time between the moment of placing an order and actually receiving the order is referred to as lead time.

When the need for the material is felt and an order is placed, it may be delivered instantaneously or it may require some time before delivery is effected. In general, lead time has four components, viz., administrative lead time, supplier's lead time, transportation lead time and inspection lead time.

v) Safety (or Buffer) Stock

In real-life situations, firms operate under the condition of uncertainty relative both to demand as well as procurement time. Total actual demand may be more or less than the forecast demand. Similarly, actual procurement time may vary from estimated time. In order to minimize uncertainty in demand and/or lead time, a firm maintains safety/buffer stock.

The safety stock may be defined as "*the minimum additional inventory to serve a safety margin or cushion to meet an unanticipated increase in usage resulting from various uncontrollable factors.*"

vi) Re-order Level (ROL)

ROL is the amount of stock that is on hand at the time of the placement of the replenishment order. This level should be enough to serve reasonably well the customers during the lead time.

Re-order Level = Safety stock + Lead time demand

5.5 The Economic Order Quantity (EOQ)

Conceptually, the firms order material for raw material and finished goods inventories so that the cost of ordering too few materials is balanced against the cost of ordering too many materials on each order. *Fig.5.1* demonstrates that annual carrying/holding costs climb as the order quantity rise.

This results from the direct relationship between order quantity and average inventory level: As order quantity increases, so do average inventory levels. On the other hand, as order quantities increase, the number of orders per year declines and thus the annual ordering cost fall. Similarly, as order quantities increase, the number of times that inventory is replenished per year declines and average inventory level rises, therefore, the number of shortages likely to occur and the intensity of these shortages decline.

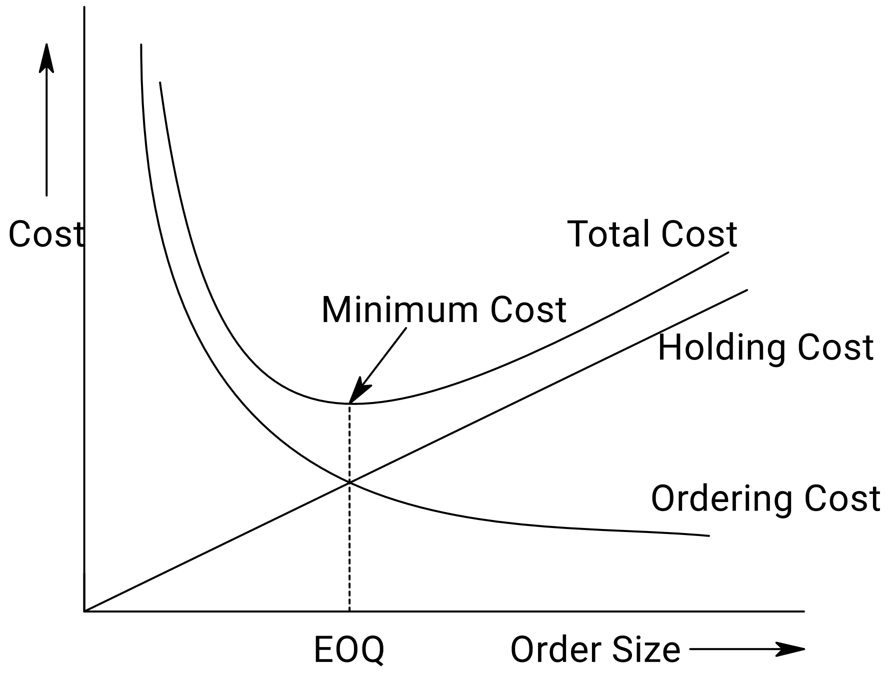


Fig.5.1 – Relationship between Economic Order Quantity and Total Costs.

Fig.5.1 shows the optimal order quantity, traditionally called the "Economic Order Quantity" (EOQ) or Economic Lot Size. *EOQ is that size of the order which minimizes total annual cost of carrying inventory and cost of ordering.*

Notations used:-

q = Number of units per order

q_O = Economic order quantity or optimal no. of units per order to minimize total cost

r = Rate of demand in units per unit time

d = Rate of arrival (replenishment rate)in units per unit time

P = Cost of 1 unit of item

C_o = Ordering (procurement or set-up) cost of each order

C_h = Holding or carrying cost per unit per unit time

t_C = Length of time between two successive orders

N = No. of orders or manufacturing runs per year

TAC = Total associated cost of inventory

5.6 Inventory Models with Deterministic Demands

It is extremely difficult to formulate a single general inventory model that takes into account all variations in the real system. In fact, even if such model were developed, it may not be analytically solvable. Thus inventory models are usually developed for some specific situations.

In this section, we shall deal with situations in which demand is assumed to be fixed and completely known. Models for such situations are called Economic Lot Size or Economic Order Quantity models.

5.6.1 Model 1: Classical EOQ Model (Demand Rate Uniform, Replenishment Rate Infinite)

This is one of the simplest inventory models. This is a deterministic, single item model i.e. it ignores all uncertainties and deals only with one item. It is characterized by

(i) Demand is constant, at a uniform rate and known with certainty,

(ii) Lead time and other system parameters such as costs are constant, independent of replenishment quantity and are known with certainty

(iii) Replenishment is instantaneous i.e. rate of replenishment is infinite and

(iv) The shortage of stock is not permitted.

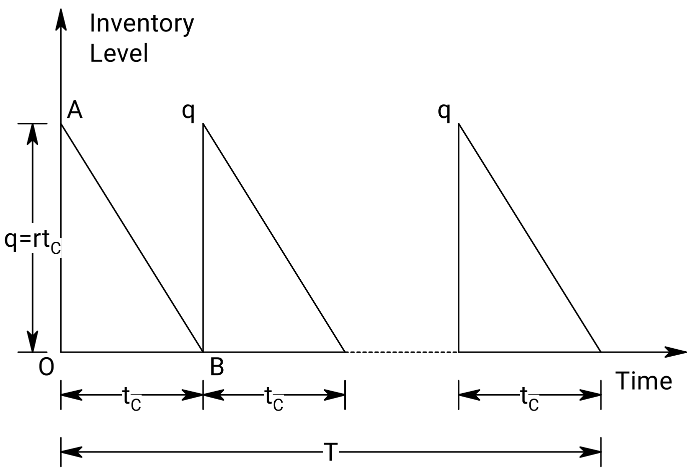


Fig.5.2 – Inventory situation for model 1

The stockist's problem is to determine

a) At which frequency he should place the order.

b) How many units should be ordered in each order?

This model is illustrated in figure **Error! Reference source not found.**

If orders are placed at intervals t, a quantity $\boldsymbol{q} = \boldsymbol{rt}$ must be ordered in each order. Since the stock in small time dt is $rtdt$, the stock in time period t will be

$$\int_0^t rtdt = \frac{1}{2}rt^2 = \frac{1}{2}qt = Area\ of\ inventory\ triangle\ OAB.$$

∴ Cost of holding inventory during time t $= \frac{1}{2}C_h rt^2$

Ordering cost to place an order $= C_o$

∴ Total cost during time t $= \frac{1}{2}C_h rt^2 + C_o$

∴ Average total cost per unit time $\mathrm{TAC(t)} = \frac{1}{2}C_h rt + \frac{C_o}{t}$ *Eq. (5.1)*

C will be minimum if $\frac{d\,TAC(t)}{dt}=0$ and $\frac{d^2\,TAC(t)}{dt^2}$ is positive.

Differentiating equation *Eq. (5.1)* with respect to 't',

$$\frac{d\,TAC(t)}{dt}=0 \Rightarrow \frac{1}{2}C_h r - \frac{C_o}{t^2} = 0,$$

$$\therefore t = \sqrt{\frac{2C_o}{C_h r}}$$

Differentiating equation *Eq. (5.1)* twice with respect to *'t'*,

$\frac{d^2TAC(t)}{dt^2} = \frac{2C_o}{t^3}$, which is positive for the value of t given by the above equation.

Thus $C(t)$ is minimum for an optimal time interval,

$$t_o = \sqrt{\frac{2C_o}{C_h r}} \qquad \textit{Eq. (5.2)}$$

Optimum quantity q_o to be ordered during each order,

$$q_o = rt_o = \sqrt{\frac{2C_0 r}{C_h}} \qquad \textit{Eq. (5.3)}$$

which is known as the optimal lot size (or economic order quantity) formula. Any other order quantity will result in higher cost.

The resulting minimum total associated cost per unit time

$$TAC(q_o) = \frac{1}{2}C_h r t_0 + \frac{C_o}{t_o}$$

$$TAC(q_o) = \frac{1}{2}C_h r\sqrt{\frac{2C_0}{C_h r}} + C_o\sqrt{\frac{C_h r}{2C_o}}$$

$$TAC(q_o) = \frac{1}{\sqrt{2}}\sqrt{C_0 C_h r} + \frac{1}{\sqrt{2}}\sqrt{C_0 C_h r} = \sqrt{2C_0 C_h r} \qquad \textit{Eq. (5.4)}$$

This shows that for the optimum value of q_o when TAC is minimum, the cost-component due to holding of inventory is equal to the cost component due to reordering. Also the total minimum coast per unit time including the cost of the item

$$= \sqrt{2C_0 C_h r} + P \cdot r$$

where P is the cost/unit of the item.

The average inventory is $\frac{q_o + 0}{2} = \frac{q_o}{2}$ and is, time-independent.

It may be realized that some of the assumptions made are not satisfied in actual practice. For instance, it is seldom that customer demand is known exactly and that replenish time is negligible.

5.6.1.1 Assumptions in E.O.Q. Formula

Following simplifying assumptions have been made while deriving the economic order quantity formula:

1. Demand is known and uniform (constant).
2. Shortages are not permitted; as soon as the stock level becomes zero, it is instantaneously replenished.
3. Replenishment of stock is instantaneous or replenishment rate is infinite.
4. The lead time is zero. The moment the order is placed, the quantity ordered is received.
5. Inventory carrying cost and ordering cost per order remain constant over time. The former is linearly related to the quantity ordered and the letter ti the number of orders.
6. The cost of the item remains constant over time. There are no price-breaks or quantity discounts.

7. The item is purchased and replenished in lots or batches.
8. The inventory system pertains to a single item.

5.6.2 Model 2: EOQ Model with Finite Replenishment Rate (Demand Rate Uniform, Replenishment or Production Rate Finite)

This model similar to that of Model 1, also assumes deterministic conditions with the only difference that supply is gradual (finite rate of arrival say 'd' units per unit time) rather than instantaneous. Obviously, when rate of arrival is more than rate of usage, inventory builds up. This situation is graphically represented in *Fig.5.1Fig.5.3*. it is described as producer's situation when source of replenishment is equivalent to manufacturing of the item.

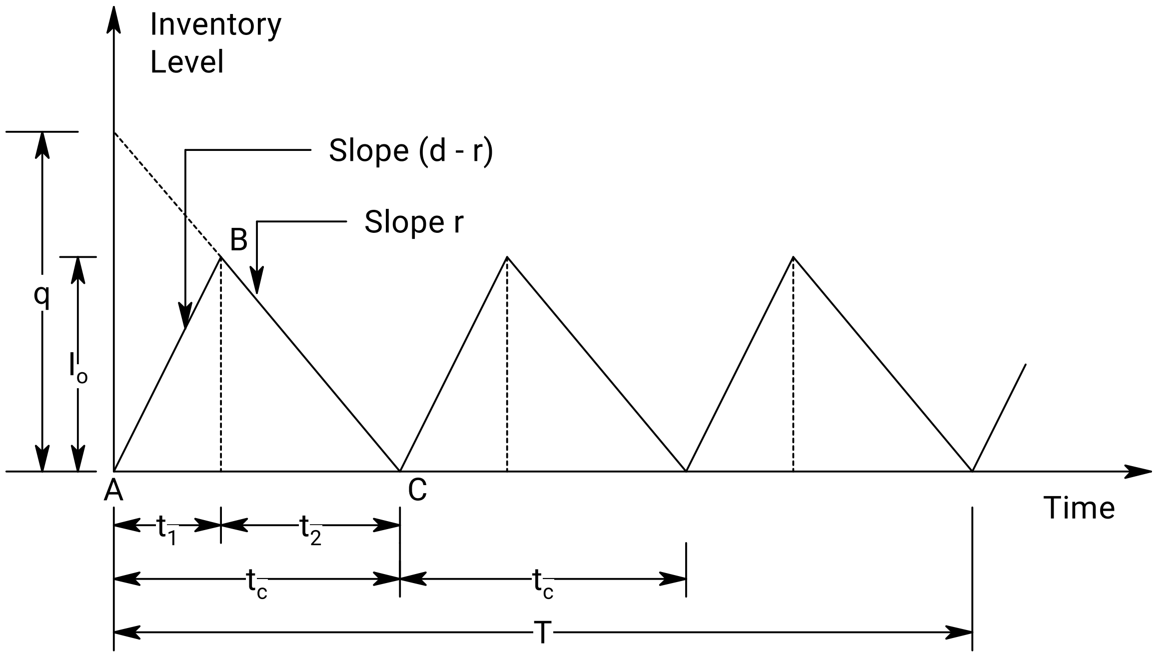


Fig.5.3 – Inventory situation with finite rate of production

From A to B there is simultaneous arrival of material (at the rate of d units per unit time) and usage of material (at the rate of r units per unit time) i.e. build-up of inventory at the rate of (d−r) units per unit time.

At point B arrival of material or the production has ceased, and material is now used from built-up inventory only. Alternatively, slop r the demand rate is extended up to beginning of the cycle gives quantity of material q that when replenished instantaneously would meet the requirement of cycle time t_c. This gives the relationship $t_c = {}^{q}/_{r}$.

5.6.2.1 Important Formulas

Maximum inventory level = Inventory accumulation rate x Period of delivery $= (d - r)\frac{q}{d}$

Minimum inventory level $= 0$

Average inventory level $= \frac{1}{2}$(Maximum inventory level + Minimum inventory level)

$$= \frac{1}{2}\left[(d - r)\frac{q}{d} + 0\right] = \frac{q}{2}\left(\frac{d - r}{d}\right)$$

Annual inventory carrying (or holding) cost per unit time = Average inventory level × Carrying cost

$$= \frac{q}{2}\left(\frac{d - r}{d}\right)C_h$$

Re-order cost or setup cost per unit time $= \frac{C_o}{t_c}$

Combining both the costs, total associated cost per unit time

$$TAC = \frac{q}{2}\left(\frac{d-r}{d}\right)C_h + \frac{C_o}{t_c}$$

Substituting $t_c = q/r$

$$TAC = \frac{q}{2}\left(\frac{d-r}{d}\right)C_h + \frac{C_o r}{q}$$

Differentiating the above equation with respect to q and equating to zero,

$$\frac{d(TAC)}{dq} = \frac{C_h}{2}\left(\frac{d-r}{d}\right) - \frac{C_o r}{q^2} = 0 \Rightarrow q_0 = \sqrt{\frac{2C_0}{C_h}\frac{rd}{(d-r)}}$$

$$\therefore \boldsymbol{EOQ = q_0 = \sqrt{\frac{2C_0 r}{C_h}} \times \sqrt{\frac{d}{(d-r)}}} \qquad \text{Eq. (5.5)}$$

When the replenishment rate d is infinite the model would be identical to model 1.

Optimum cycle time

$$t_{co} = \frac{q_0}{r} = \sqrt{\frac{2C_o d}{C_h(d-r)r}}$$

The minimum total associated cost of inventory

$$(TAC)_{min} = \frac{q_0}{2}\left(\frac{d-r}{d}\right)C_h + \frac{C_o r}{q_0}$$

Putting the value of q_0 in above equation

$$(TAC)_{min} = \frac{C_h(d-r)}{2d}\sqrt{\frac{2C_0 r}{C_h}} \times \sqrt{\frac{d}{(d-r)}} + C_o r\sqrt{\frac{C_h(d-r)}{2C_o rd}}$$

$$= \sqrt{\frac{C_0 C_h(d-r)r}{2d}} + \sqrt{\frac{C_0 C_h(d-r)r}{2d}}$$

$$\boldsymbol{(TAC)_{min} = \sqrt{2C_0 C_h r} \times \sqrt{\frac{(d-r)}{d}}}$$

Optimal number of production runs per year:

$$N = \frac{r}{q_o}$$

Optimal length of each production lot size production run:

$$t_1 = \frac{q_o}{d}$$

5.6.3 Model 3: Demand rate uniform, Replenishment Rate Infinite, Shortages Allowed

In the models discussed earlier, a shortage (or stock out) is considered undesirable and is avoided, if possible. This is because shortages, and are likely to, mean loss of customer goodwill, reduction in future orders, unfavourable change in the market share, and so on. While in some situations the customer shift to other sources for their requirements, and so may be lost forever, in some other customers may not withdraw the orders and wait until the next shipment arrives. This latter situation is called the back-order situation.

This model has the same characteristics as that of model 1, viz. demand is known and is at uniform rate and replenishment of inventory is instantaneous. Further, it is stipulated that the shortage of inventory is permitted at finite cost. In the earlier model shortage was not permitted. In other words, the cost of shortage then was infinite. In this model, for the production system there is an associated cost of not having the material in the stock; for a supply system this means penalty clause for delayed delivery. The graphical situation of this model is shown in *Fig.5.4*.

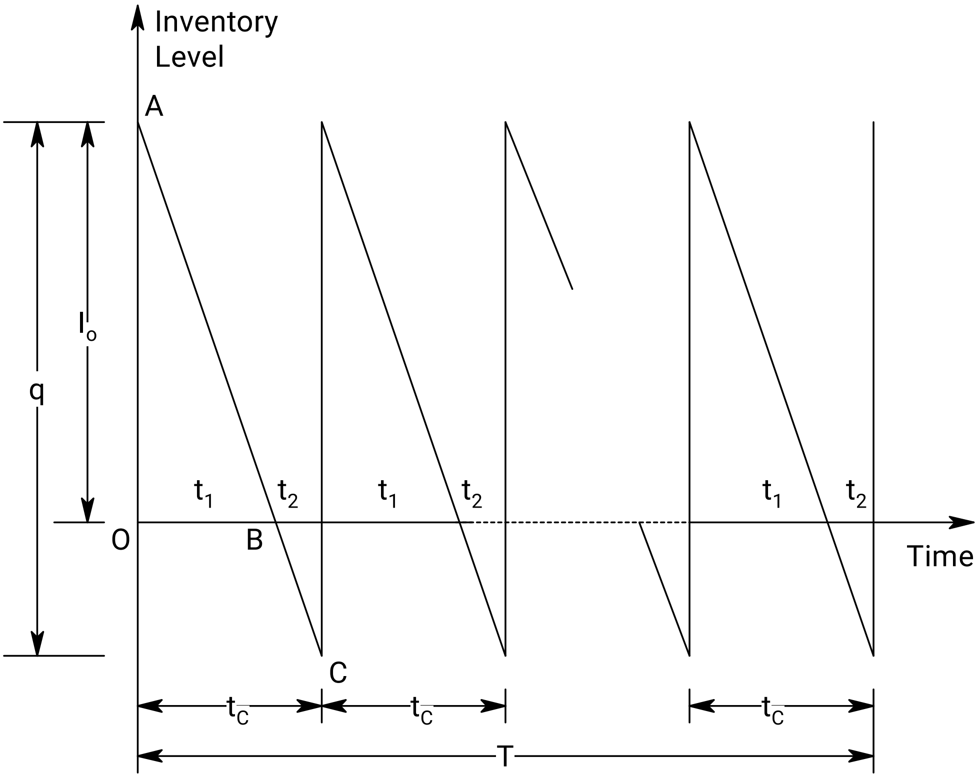


Fig.5.4 – Inventory situation for model 3

Let, t_1 be the period of the cycle during which inventory is positive

t_2 be the period of the cycle during which inventory is negative

q be the ordering quantity

I_O be the stock at the beginning of the cycle, i.e. net inventory level.

Total associate cost for ordering q, with initial stock I_O and cycle time t_C per cycle.

$TAC\ (q, I_O) =$ Cost of carrying inventory + Cost of shortage + Cost of re-order

During each cycle inventory is carried during time t_1, and the average amount is $\frac{I_O}{2}$.

Therefore the cost of carrying inventory is $\frac{C_h I_O t_1}{2}$.

During each cycle shortage of inventory is during time t_2, and the average shortage is $\frac{(q - I_O)}{2}$.

Therefore the cost of shortage of inventory is $\frac{C_s(q - I_O)t_2}{2}$.

Therefore,

$$TAC\ (q, I_O) = \frac{C_h I_O t_1}{2} + \frac{C_s(q - I_O)t_2}{2} + C_O\ per\ cycle.$$

and, $$TAC\ (q, I_O) = \frac{C_h I_O t_1}{2t_C} + \frac{C_s(q - I_O)t_2}{2t_C} + \frac{C_O}{t_C}\ per\ cycle. \qquad \text{Eq. (5.6)}$$

Referring *Fig.5.4* and using the relationship of similar triangles,

$$\frac{t_1}{t_C} = \frac{I_O}{q} \Rightarrow t_1 = \frac{I_O \cdot t_C}{q}$$

$$\frac{t_2}{t_C} = \frac{q - I_O}{q} \Rightarrow t_2 = \frac{q - I_O}{q} t_C \qquad and \qquad t_C = \frac{q}{r}$$

Putting these values in *Eq. (5.6)*.

$$TAC\ (q, I_O) = \frac{C_h}{2} \cdot \frac{{I_O}^2}{q} + \frac{C_s}{2} \cdot \frac{(q - I_O)^2}{q} + \frac{C_O r}{q}$$

For maximum or minimum, as TAC is a function of two variables *q* and *I*, it can be partially differentiated with respect to *q* and *I* and partial differentials equated to zero to yield relation between model parameters.

$$\frac{\partial(TAC)}{\partial I_O} = \frac{\partial}{\partial I_O}\left(\frac{C_h}{2} \cdot \frac{{I_O}^2}{q} + \frac{C_s}{2} \cdot \frac{(q - I_O)^2}{q} + \frac{C_O r}{q}\right) = \frac{C_h I_O}{q} - \frac{C_S(q - I_O)}{q} = 0$$

$$\therefore \frac{I_O}{q - I_O} = \frac{C_S}{C_h}$$

$$\therefore \boldsymbol{I_O = q\frac{C_S}{C_S + C_h}} \qquad \text{Eq. (5.7)}$$

The equation *Eq. (5.7)* gives net inventory level (or maximum inventory level) at the beginning of cycle as a fraction of quantity ordered.

Now we have, $\frac{I_O}{q - I_O} = \frac{C_S}{C_h}$

$$\therefore q - I_O = I_O \frac{C_h}{C_s}$$

But $q - I_O =$ Shortage Permitted $=$ S and putting the value of I_O from *Eq. (5.7)*, we get

$$\therefore \boldsymbol{S = q\frac{C_h}{C_S + C_h}} \qquad \text{Eq. (5.8)}$$

The above equation stands for maximum shortage permitted in the model as fraction of ordering quantity.

Further,

$$\frac{\partial(TAC)}{\partial q} = \frac{\partial}{\partial q}\left(\frac{C_h}{2} \cdot \frac{{I_O}^2}{q} + \frac{C_s}{2} \cdot \frac{(q - I_O)^2}{q} + \frac{C_O r}{q}\right) = -\frac{C_h {I_O}^2}{2q^2} + \frac{C_S}{2}\frac{2q(q - I_O) - (q - I_O)^2}{q^2} - \frac{C_O r}{q^2} = 0$$

Simplifying above equation we get,

$$\frac{C_S q^2}{2} - \frac{(C_h + C_S)}{2}{I_O}^2 - C_O r = 0$$

Putting the value of I_O from *Eq. (5.7)*,

$$\frac{C_S q^2}{2} - \frac{(C_h + C_S)}{2}\left(\frac{C_s}{C_h + C_S}\right)^2 q^2 - C_O r = 0$$

Simplifying, we get EOQ,

$$q_O = \sqrt{\frac{2C_O r}{C_h}} \cdot \sqrt{\frac{C_h + C_S}{C_s}}$$

Similarly the minimum total associated cost will be

$$TAC_{min} = \sqrt{2C_0 C_h r} \cdot \sqrt{\frac{C_s}{C_h + C_S}}$$

5.6.4 Model 4: Demand rate uniform, Replenishment (or Production) Rate Finite, Shortages Allowed

This model has the same assumptions as in model 3 except that the production rate is finite. *Fig.5.4* shows the variation of inventory with time.

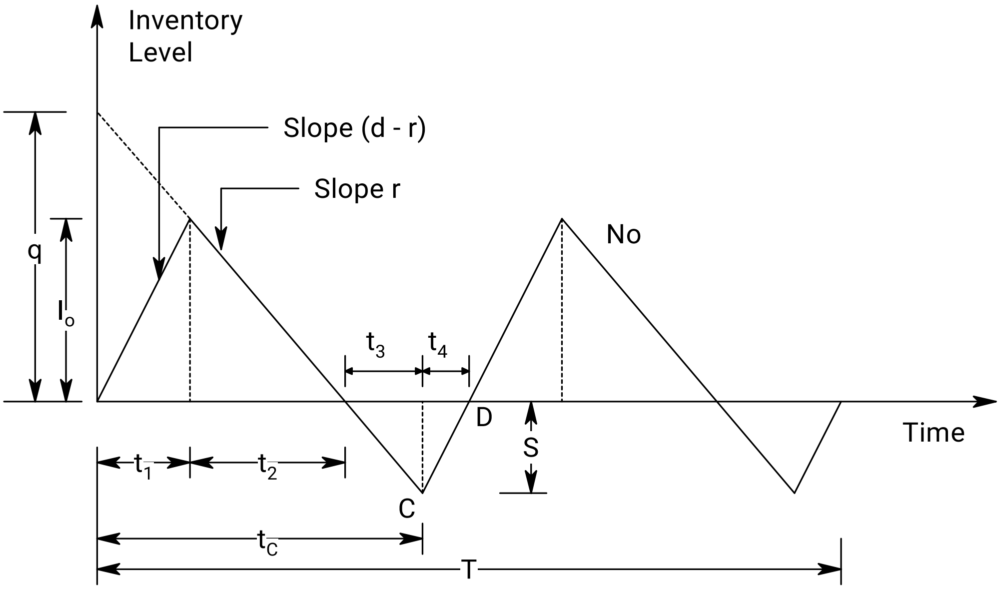


Fig.5.5 – Inventory situation for model 4

Referring to *Fig.5.5*, we find that inventory is zero in the beginning. It increases at a constant rate (d – r) for time t_1 until it reaches a level I_O. There is no replenishment during time t_2, inventory decreases at constant rate *r* till it becomes zero. Shortage starts increasing at constant rate *r* during time t_3 until this backlog reaches a level S. Lastly, production starts and backlog is filled at a constant rate (d – r) during time t_4 till the backlog becomes zero. This completes one cycle.

5.6.4.1 Important Formulas

a. Economic Order Quantity

$$q_O = \sqrt{\frac{2C_O r}{C_h}} \cdot \sqrt{\frac{C_h + C_S}{C_s}} \cdot \sqrt{\frac{d}{d - r}}$$

b. Maximum Inventory Level

$$I_O = q_O \cdot \frac{C_S}{C_S + C_h} \cdot \frac{d - r}{d}$$

c. Maximum Shortage Permitted

$$S = q_O \cdot \frac{C_h}{C_S + C_h} \cdot \frac{d - r}{d}$$

d. Minimum Total Associated Cost

$$TAC_{min} = \sqrt{2C_0C_hr} \cdot \sqrt{\frac{C_s}{C_h + C_s}} \cdot \sqrt{\frac{d-r}{d}}$$

5.6.5 EOQ Model with Quantity Discounts

Quantity discounts occur in numerous situations where suppliers provide an incentive for large order quantities by offering a lower purchase cost when items are ordered in larger lots or quantities. In this section, we will see how the EOQ model can be used when quantity discounts are available.

Following formulas will be used in this model:

EOQ without discounts

$$q_O = \sqrt{\frac{2C_O r}{C_h}}$$

EOQ with discounts

$$Q_O = \frac{C_d \cdot r + q_O \cdot i \cdot P}{i(P - C_d)}$$

Max Net Saving with discounts

$$X_{max} = \frac{(C_d \cdot r + q_O \cdot i \cdot P)^2}{2ir(P - C_d)} - C_O$$

5.7 Probabilistic Inventory Model

The models discussed in previous sections are only artificial since in practical situations demand is hardly known precisely. In most situations, demand is probabilistic since only the probability distribution of future demand, rather than the exact value of demand itself is known. The probability distribution of future demand is usually determined from the data collected from past experience.

Expected costs are obtained by multiplying the actual costs for a particular situation with the probability of occurrence of that situation and then either summing or integrating according, as the probability distribution is discrete or continuous.

Costly spare parts, perishable goods, seasonal items, and fashion goods are examples of probabilistic models. Replacement orders are either not possible or become abnormally expensive and uneconomical. The decision is of the one-shot type.

The single-period inventory model refers to inventory situations in which one order is placed for the product; at the end of the period, the product has either sold out, or there is a surplus of unsold items that will be sold for a salvage value. The single-period inventory model is applicable in situations involving seasonal or perishable items that cannot be carried in inventory and sold in future periods.

Seasonal clothing (such as bathing suits and winter coats) is typically handled in a single-period manner. In these situations, a buyer places one preseason order for each item and then experiences a stock out or hold a clearance sale on the surplus stock at the end of the season. No items are carried in inventory and sold the following year.

Newspapers are another example of a product that is ordered one time and is either sold or not sold during a single period. While newspapers are ordered daily, they cannot be carried in inventory and sold in later periods. Thus, newspaper orders may be treated as a sequence of single-period models; that is, each day or period is separate, and a single-period inventory decision must be made each period (day). Since we order only once for the period, the only inventory decision we must make is how much of the product to order at the start of the period. Because newspaper sales are an excellent example of a single-period situation, the single-period inventory problem is sometimes referred to as ***the newsboy problem***.

Optimum stock level

$$P_{r1} < \frac{C_S}{C_S + C_h} < P_{r2}$$

5.8 ABC Analysis

This system of control is also known as the Selective Approach System. This is sometimes known as Always Better Control. In ABC system of inventory control, the materials are classified depending on their turnover and annual consumption cost.

A - Class Items

These items are less in number, but consumes a large portion of the total inventory investment. Here annual consumption cost is important than the unit cost of the material. For example let us consider, two materials Material X and material Y. The unit cost of material X is Re.1/- and annual consumption is 1000 units. The unit cost of material Y is Rs.200 and the annual consumption is 3 units. Then annual consumption cost of material X is Rs.1000/- and that of Y is Rs. 600/-. Here Material X is considered as high consumption cost material than Y. Like that in any industry, we may find that there will be certain items that are few in number but they consume nearly 70 % of inventory cost. Such items are classified as 'A' - class items.

B - Class Items

There will be certain materials, whose total annual consumption cost will be somewhere in between 20 to 25 % of total inventory investment. These items are labeled as 'B' - class items. These items will form 60 percent of number of items stored.

B - Class Items

The last class of items that are labeled as 'C' - class items, will be large in number maybe 30 to 35 % of total number of items stored, but consumes only 5 to 10 % total inventory investment.

Hence we can say that A-Class items are less in number and consume more money, B - Class items are medium in number and consume 20 to 25 % inventory investment and C - Class items are large in number and consume only 5 to 10 percent of inventory investment. This can be shown by means of a graph as shown in *Fig.5.6*.

ABC rule can be applied to any situation where selective classification is possible. **The point to remember here is ABC analysis depends on annual consumption cost and not on the unit cost of material.**

'A' class items need the attention of higher officials and demand extreme control regarding the cost.

As they consume 70% of money even 10 % saving through bargaining or inventory control techniques, the savings will be worthwhile. 'B' Class items require the attention of middle-level managers as they consume 20 to 25 % of investment on inventory. Whereas 'C' class items are left to the control of lower officials.

Procedure for ABC analysis

1. List out all items in stores along with their unit price and annual consumption.

2. Calculate the annual consumption cost of each item, which is given by multiplying the quantity consumed in the time period and the unit cost. If ‘q’ is the quantity consumed in the time period and ‘p’ is the unit price then annual consumption value **= q × p = qp.**
3. Rearrange the list in the descending order of the annual consumption cost. i.e. the highest cost at the top and next highest is the second and so on and the last item is the lowest consumption value item.
4. Calculate the cumulative total of annual consumption value.
5. Find the parentage of each cumulative value with respect to the total cost of inventory.
6. Mark a line at 70%, 90% and at 100%. All the items covered by 70% line are ‘A’ class items, those which are covered between 70% line and 90% line are ‘B’ class items and those are covered by 90% and 100 % are ‘C’ class items.

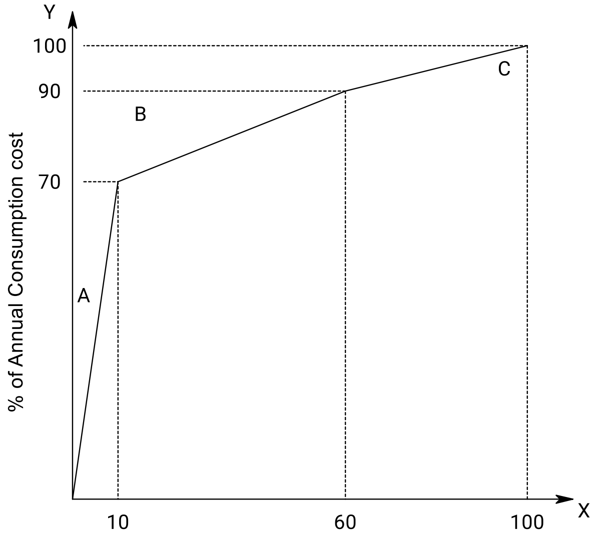


Fig.5.6 – ABC Graph

5.9 Class Examples

Ex. 5.1 A supplier is required to deliver 20000 tons of raw materials in one year to a large manufacturing organization. The supplier maintains his go-down to store the material received from various resources. He finds that cost of inventory holding is 30 paisa per ton per month. His cost for ordering the material is Rs. 400. One of the conditions of the supplier contract from the manufacturing organization is that the contract will be terminated in the event of supply not being maintained as a schedule. Determine (1) in what lot size is the supplier should produce the material for minimum total associated cost of inventory? (2) At what time interval should he procure the material? It may be assume that replacement of inventory is instantaneous.

Solution:

Ex. 5.2 In example 5.1, if there is (i) 10 per cent increase in holding cost or (ii) 10 percent increase in ordering cost, in each case determine the optimal lot size and corresponding minimum total expected cost of inventory. Comment the result.

Solution:

Ex. 5.3 A certain item costs Rs. 250 per ton. The monthly requirement is 5 tons and each time the stock is replenished, there is an order cost of Rs. 120. The cost of carrying inventory has been estimated at 10% of the value of the stock per year. What is the optimal order quantity? If lead time is 3 months, determine the re order point. At what intervals the order should be placed?

Solution:

Ex. 5.4 A manufacturer has to supply his customers with 1200 units of his product per annum. The inventory carrying cost amounts to ₹ 1.2 per unit. The set-up cost per run is ₹ 160. Find:

i) EOQ
ii) Minimum average yearly cost
iii) Optimum no of orders per year
iv) The optimum time between orders (optimum period of supply per optimum order)

Solution:

Ex. 5.5 The annual demand per item is 6400 units. The unit cost is ₹ 12 and the inventory carrying charges 25% per annum. If the cost of procurement is ₹ 300 determine:

I. EOQ
II. No. of orders per year
III. Time between 2 consecutive orders
IV. Optimum cost

Solution:

Ex. 5.6 A manufacturing company needs 4000 units of material every month. The delivery system from the supplier is so scheduled that once delivery commences the materials is received at the rate of 6000 units per month. The cost of processing purchase order is Rs. 600 and the inventory carrying cost is 30 paisa per unit per month. Determine the optimal lot size and interval at which the order is to be placed. What is maximum inventory during a cycle?

Solution:

Ex. 5.7 The demand for a certain item is 150 units per week. No shortages are to be permitted. Holding cost is 5 paisa per unit per week. Demand can be met either by manufacturing or purchasing. With each source the data are as follows:

	Manufacture	Purchase
Item cost in Rs. per unit	10.50	12
Set up/ Ordering cost in Rs. per order/setup	90	20
Replenishment rate in units per week	260	Infinite
Lead time in weeks	4	10

Determine (a) the minimum cost procurement source and its economic advantage over its alternative resource, (b) E.O.Q. or E.B.Q. as per the source selected, (c) the minimum procurement level (Re-order point)

Solution:

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Ex. 5.8 A tractor manufacturing company has entered in to a contract with M/s Auto Diesel for delivering 30 engines per day. M/s Auto Diesel has committed that for every day's delay in delivery; there will be penalty of delayed supply at the rate of Rs. 100 per engine per day. M/s Auto Diesel has the inventory holding cost of Rs. 600 per engine per month. Assume replenishment of engines as instantaneous and ordering cost as Rs. 15000. What should be initial inventory level and what should be ordering quantity for minimum associated cost of inventory? At what interval procurement should be made?

Solution:

Ex. 5.9 In above example, find out optimum order quantity if shortage is not permitted. Compare this with the value of obtained in above example and comment on the result.

Solution:

Ex. 5.10 The demand for an item in a company is 18000 units per year and the company can produce the item at a rate of 3000 units per month. The set up cost is Rs. 500 per set up and the annual inventory holding cost is estimated at 20 percent of the investment in average inventory. The cost of one unit short is Rs. 20 per year. Determine, (i) Optimal production batch quantity, (ii) Optimum cycle time and production time, (iii) Maximum inventory level in the cycle, (iv) Maximum shortage permitted and (v) Total associated cost per year. The cost of the items is Rs. 20 per unit.

Solution:

Ex. 5.11 A wholesale dealer in bearings purchases 30000 bearings annually at intervals and order size suitable to him. The price is Rs. 150 per bearing. The manufacturing company offers the dealer a discount of Rs. 7 per bearing for the order size larger than earlier. The reorder cost is Rs. 40 and the inventory carrying cost amounts to 20 percent of the investment in purchase price. Decide the optimum order size for special discount offer purchase and the maximum benefit he can derive from this order.

Solution:

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Ex. 5.12 A large industrial campus has decided to have its diesel generator system for street lighting, security illumination and round the clock process systems. The generator needs a tailor made for each other control unit which cost Rs. 18000 per number when ordered with the total equipment of diesel generator. A decision needs to be taken whether additional numbers of this unit should be ordered along with equipment, and if so, how many units should be ordered? These control units, though tropicalized and considered quite reliable, are known to have failed from time to time and history of failures of similar equipment give the following probability of failure.

No. of units having failed and hence No. of spare required	0	1	2	3	4	5	6
Probability	0.6	0.2	0.1	0.05	0.03	0.02	0

It is found that if the control unit fails, the entire generator system comes to a grinding halt. When control unit fails and a spare unit is not available it is estimated that the cost rush order procurement, including the associated cost of the downtime is Rs. 50000 per unit. Considering that any investment in inventory is the cost of inventory, decide how many spare units should be ordered along with the original order. Determine total associated cost for each no. of spare unit.

Solution:

Ex. 5.13 In the above problem as regular purchase price of control unit is almost one third of the estimated rush order associated cost of one unit. The management decides to buy two spare units with the first order. Having decided that, the management would like to know for what range of actual values of shortage cost, the decision is justified.

Solution:

Ex. 5.14 Probabilistic demand of sweets in a large chain of sweet marts is rectangular between 1000 kg and 1400 kg. Profit per kg of fresh sweet sold is Rs. 14.70. If sweet is not sold fresh, next day it can be sold at a loss of Rs. 2.30 per kg. Determine the optimum stock to have fresh sweet on hand every day.

Solution:

Ex. 5.15 A newspaper boy buys daily papers from vendor and gets commission of 4 paisa for each paper sold. As he is always demanding large number in a lot, he has agreed to pay 3 paisa per each copy returned unsold. He has the past experience of the demand (its probability) as under.

23 (0.01), 24 (0.03), 25 (0.06), 26(0.10), 27(0.20), 28(0.25), 29(0.15), 30(0.10), 31(0.05), 32(0.05)

How many papers should he lift from vendor for minimum associated cost?

Solution:

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5.10Lab Examples

Ex. 5.16 M/s T.V. assembly, one man T.V. assembler – entrepreneur, needs 10000 of tubes per year. The cost of one procurement is Rs. 80. The holding cost per tube is Rs. 3 per year. The rush purchase of tubes, if not in stock, amounts to equivalent shortage cost of Rs. 6 per tube per year. If stock ordered is delivered all instantaneously, determine how much he should order, at what interval and what will then be the total associated cost of inventory?

Solution:

Ex. 5.17 In above example, M/s T.V. assembly seeks to reduce holding cost to Rs. 2.4 per tube per annum, and with patronized supplier manages to reduce procurement cost to Rs. 60 per order. What percentage reduction in penalty cost for shortage should negotiates that his total associated cost of inventory is reduced by 50 percent?

Solution:

Ex. 5.18 A manufacturer requires 15000 units of a part annually for an assembly operation. He can produce this part at the rate of 100 units per day. The set up cost for each production run is Rs. 50. To hold one unit of this part in inventory costs Rs. 5 per year. Shortage cost is Rs. 15 per unit per year. Cost of the part is Rs. 20 per unit. Assuming 250 working days per year, what will be the optimum manufacturing quantity? What will be the time between two production runs? What will be the total annual cost of the inventory system?

Solution:

Ex. 5.19 A large scale truck fleet operator has to supply truck at the rate of 30 every day. If on account of prolonged repairs and maintenance he is not able to supply the trucks, he has to incur the cost of short supply, loss of profit at the rate of Rs. 100 per day per truck. On the other hand, if he has road worthy trucks in excess of requirements he has to incur holding cost of Rs. 20 per day per truck. Every time he orders the lot of truck from repairs department, he incurs the cost of Rs. 40 per order. How many trucks should be ordered from repair department at a time, and at what interval should he put the order? What is the total associated cost of inventory then?

Solution:

Ex. 5.20 Farm equipment manufacture undertakes to have a transshipment delivery of 40 trailers every day in a huge construction plant. His short supply results in the loss of Rs. 40 per unit per day. In case he has more trailers on hand then required he has to incur the cost of Rs. 5 p trailer per day. If he makes it a policy to receive the delivery at the fixed interval of 1 month, how much he should order and what should be his stock at the beginning of the month?

Solution:

Ex. 5.21 A newspaper stall sells feature magazine at a sale commission of Rs. 2 per copy. The sale is a probabilistic rectangular distribution between 500 to 600 copies. If the vendor has decided to book 580 copies what price reduction he must be thinking to offer for the sale of old issues?

Solution: