6

Replacement Theory

Contents

6.1 Introduction

Replacement theory is concerned with the problem of replacement of machines, equipments, men, etc. due to change in their performance, deteriorating efficiency, failure or breakdown. This change may either be gradual or all of a sudden.

Broadly speaking, the requirement of a replacement may be in any of the following situations:

i. An item fails and does not work at all or the item is expected to fail shortly.
ii. An item deteriorates and needs expensive maintenance.
iii. A better design of the equipment is available.
iv. It is economical to replace equipment in anticipation of costly failure.

When studying the problem of replacement, we may or may not consider the time value of money.

6.2 Failure Mechanism of Items

The word failure has got a wider meaning in industrial maintenance than what it has in our daily life. We can categorize the failure as follows:

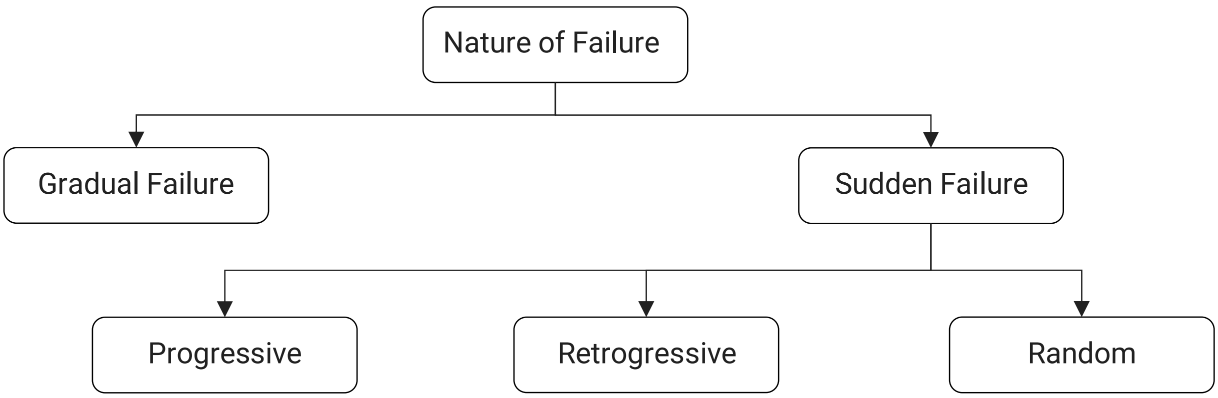

Fig.6.1 – Classification of Failure

I) Gradual failure:

The failure mechanism under gradual failure is progressive i.e., as the life of an item increases, its efficiency deteriorates resulting in:

(a) Progressive increase in maintenance expenditure or operating costs,
(b) Decreased productivity of the equipment and
(c) A decrease in the value of the equipment i.e. the resale value of the equipment/facility decreases.

Examples of this category are automobiles, bearings, machine tools, etc.

II) Sudden failure:

In this case, the items ultimately fail after a period of time. The life of the equipment cannot be predicted and is some sort of random variable. The period between installation and failure is not constant for any particular type of equipment but will follow some frequency distribution, which may be:

(a) Progressive failure:

In this case probability of failure increases with the increase in the life of an item. The best example is electrical bulbs and computer components. It can be shown as in *Fig.6.2*.

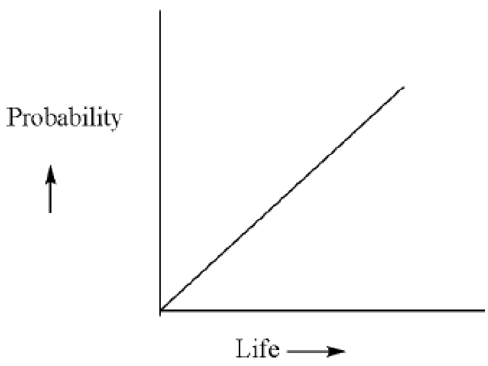

Fig.6.2 – Progressive failure

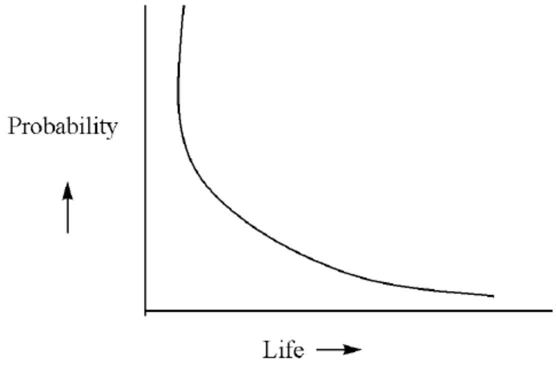

Fig.6.3 – Retrogressive failure

(b) *Retrogressive failure:*

Some items will have a higher probability of failure at the beginning of their life, and as the time passes chances of failure becomes less. That is the ability of the item to survive in the initial period of life increases its expected life. The examples are newly installed machines in production systems, new vehicles, etc. This can be shown as in *Fig.6.3*.

(c) *Random failure:*

In this class, constant probability of failure is associated with items that fail from random causes such as physical shocks, not related to age. In such cases, all items fail before aging has any effect. This can be shown as in *Fig.6.4*. An example is vacuum tubes.

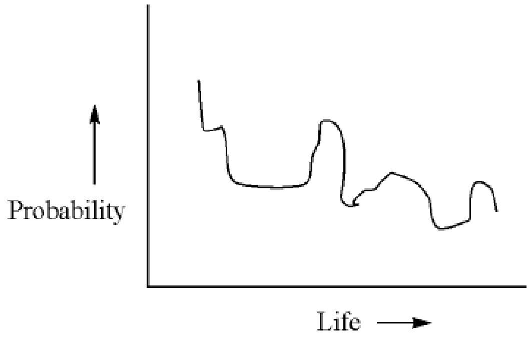

Fig.6.4 – Random failure

6.3 Costs Associated with Maintenance

Our main aim in this chapter is to find optimal replacement period so as to minimize the maintenance cost. Hence we are very much interested in the various cost associated with maintenance. Various costs to be discussed are:

I) Purchase cost or Capital cost (C)

This cost is independent of the age of the machine or usage of the machine. This is incurred at the beginning of the life of the machine, i.e. at the time of purchasing the machine or equipment. But the interest on the invested money is an important factor to be considered.

II) Salvage value / Scrap value / Resale value / Depreciation (S)

As the age of the machine increases, the resale value decreases as its operating efficiency decreases and the maintenance costs increases. It depends on the operating conditions and the life of the machine.

III) Running costs including maintenance, Repair and Operating costs (M_t)

These costs are the functions of the age of the machine and the usage of the machine. As the usage increases or the age increases, due to wear and tear, many components fail to work and they are to be replaced. As the age increases, failures also increase and the maintenance costs go on increasing. At some period the maintenance costs are so high, which will indicate that the replacement of the machine or equipment is essential.

6.4 Replacement of an Item Which Deteriorates Gradually with Time

The various techniques we may come across to analyze the situation are:

a) Replacement of items whose running cost increases with time and value of money remains the same during the period,
b) Replacement of items whose running cost increases with time and the value of money also changes with time, and

6.4.1 Replacement of Items whose Running Cost Increases with Time and the Value of Money Remains Same During a Period

We know that the cost of a piece of equipment over a given time period, say n years, has three elements:

I) Purchase Cost
II) Value Remaining after n years
III) Maintenance cost for n years

Let, $C =$ the purchase cost of the product,

$S =$ the scrap value of the equipment at the end of n years, and

$M_t =$ the maintenance cost of the equipment in year t.

The total cost, $T(n)$, of owning and maintaining the equipment for n years shall be

$$T(n) = C - S + \sum_{t=1}^{n} M_t \qquad \text{Eq. (6.1)}$$

Correspondingly, the average cost, $A(n)$, would be defined as,

$$A(n) = \frac{1}{n}\left[C - S + \sum_{t=1}^{n} M_t\right] \qquad \text{Eq. (6.2)}$$

Present worth factor:

To understand present worth factor of money lets see one example:

Suppose a businessman borrows money for his initial investment and working capital from various sources, he has to pay interest for the money he has borrowed. The amount of interest he has to pay depends on the rate of interest and the period for which he has borrowed (that is the period in which he has repaid the amount borrowed). The borrowed money is known as ***Principal (P)***, and the excess amount he has paid is known as ***Interest (i)***. The sum of both principal and the interest is known as ***Amount (A)***.

If P is the principal, 'i' is the rate of interest, and A is the amount, then the amount at the end of 'n' years with compound interest is:

$$\boldsymbol{A = P(1+i)^n} \quad \Rightarrow \quad \boldsymbol{P = (A)/(1+i)^n} \quad \Rightarrow \quad \boldsymbol{P = A \times Pwf}$$

Where Pwf is Single payment present worth factor. It is represented by **'d'** and is also known as discount rate. Discount rate is always less than one. In other words we can say that **'d'** or the present worth factor (Pwf) is present value of one rupee spent after 'n' years from now. Hence P is known as the present worth of an amount A paid in 'n' years at interest rate 'i'.

6.4.2 Replacement of Items whose Running Costs Increases with Time and Value of Money also Changes with Time

The maintenance cost goes on increasing with usage or age or time and then we have to find out the optimum time of replacing the item. Here the value of money decreases with a constant rate which is known as its depreciation ratio or discounted factor and is represented by ***d***.

Here the money value changes can be understood as follows:

Suppose a person borrows Rs. 1000/- at an interest rate of 10 % per year. After one year from now, he has to pay back Rs. 1100/-. That means to say today's Rs. 1000/- is equivalent to Rs. 1100/- after one year.

OR

Rs. 1100/- after one year is equivalent to Rs. 1000/- today. That is Re.1/- after one-year from now is equal to $1000 / 1100 = (1.1)^{-1}$ at present. This is known as the ***present value***.

To generalize, if the interest on Re.1/- is 'i' per year then the present value or present worth of Re. 1/- after one year from now is $1 / (1 + i)$, this is the depreciation ratio, represented by ***d***. Similarly, the present worth of Re.1/- to be spent after 'n' years from now (the rate of interest is 'i') is $\frac{1}{(1+i)^n}$.

6.5 Replacement of Items that Fail Completely and Suddenly

In previous sections, we discussed the replacement of items that deteriorate with time. In real-life situations, there are certain items or systems or products, whose probability of failure increases with time. They may work with designed efficiency throughout their life and if they fail to act they fail suddenly. The nature of these items is they are costly to replace at the same time and their failure affect the functioning of the entire system.

For example, resistors, components of the air-conditioning unit and certain electrical components. If we do not replace the item immediately, then the loss of production, idle labour; idle raw materials, etc are the results. It is evident, the failure of such items causes heavy losses to the organization. Such situations demand the formulation of a policy, which will help the organization to avoid losses. Sometimes we find it is better to replace the item before it fails so that the expected losses due to failure can be avoided. The following courses of action can be followed for the replacement of such items.

6.5.1 Individual Replacement Policy

This policy states that replace the item soon after its failure. Here the cost of replacement will be somewhat greater as the item is to be purchased individually from the seller as and when it fails. From the time of failure to the replacement, the system remains idle. More than that, as the item is purchased individually, the cost of the item may be more.

6.5.2 Group replacement policy

If the organization has got the statistics of failure of the item, it can calculate the average life of the item and replace the item before it fails, so that the system can work without break. In this case, all the items, even they are in good working condition, are replaced at a stipulated period as calculated by the organization by using the group replacement policy. One thing we have to remember is that, in case any item fails, before the calculated group replacement period, it is replaced individually immediately after failure. Hence this policy utilizes the strategy of both individual replacement and group replacement.

6.6 Class Examples

Ex. 6.1 The cost of equipment is Rs. 62,000 and its scrap value is Rs. 2,000. The life of the equipment is 8 years. The maintenance costs for each year are as given below:

Year	1	2	3	4	5	6	7	8
Maintenance Cost in Rs.	1000	2000	3500	5000	8000	11000	16000	24000

When the equipment should be replaced?

Solution:

Year n	Resale Price S	Maintenance Cost M_t	Cumulative Maintenance Cost $\sum M_t$	Total Cost $TC = C - S + \sum M_t$	Average Annual Cost $ATC = \frac{TC}{n}$

Ex. 6.2 A manufacturer finds from his past records that the costs per year associated with a machine with a purchase price of Rs. 50,000 are as given below:

Year	1	2	3	4	5	6	7	8
Maintenance Cost in Rs.	15000	16000	18000	21000	25000	29000	34000	40000
Scrap value in Rs.	35000	25000	17000	12000	10000	5000	4000	4000

What is the optimal period of replacement?

Solution:

Year n	Resale Price S	Maintenance Cost M_t	Cumulative Maintenance Cost $\sum M_t$	Total Cost $TC = C - S + \sum M_t$	Average Annual Cost $ATC = \frac{TC}{n}$

Ex. 6.3 (a) Machine A cost Rs. 36,000. Annual operating costs are Rs. 800 for the first year, and then increase by Rs. 8000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy if followed, what will be the yearly cost of owning and operating the machine?

(b) Machine B costs Rs. 40,000. Annual operating costs Rs. 1,600 for the first year, and then increase by Rs. 3,200 every year. You now have a machine of type A which is one year old. Should you replace it with B, if so when? Assume that both machines have no resale value.

Solution:

(a) Machine A

Year n	Resale Price S	Maintenance Cost M_t	Cumulative Maintenance Cost $\sum M_t$	Total Cost $TC = C - S + \sum M_t$	Average Annual Cost $ATC = \frac{TC}{n}$

(b) Machine B

Year n	Resale Price S	Maintenance Cost M_t	Cumulative Maintenance Cost $\sum M_t$	Total Cost $TC = C - S + \sum M_t$	Average Annual Cost $ATC = \frac{TC}{n}$

Ex. 6.4 The data on the operating costs per year and resale price of equipment A whose purchase price is Rs. 10,000 are given below:

Year	1	2	3	4	5	6	7
Maintenance Cost in Rs.	1500	1900	2300	2900	3600	4500	5500
Resale value in Rs.	5000	2500	1250	600	400	400	400

I) What is the optimum period of replacement?

II) When equipment A is 2 year old equipment B which is a new model for the same usage is available. The optimum period for the replacement is 4 years with an average cost of Rs. 3,600. Should we change equipment A with that of B? If so when?

Solution:

Year n	Resale Price S	Maintenance Cost M_t	Cumulative Maintenance Cost $\sum M_t$	Total Cost $TC = C - S + \sum M_t$	Average Annual Cost $ATC = \frac{TC}{n}$

Ex. 6.5 A firm pays Rs. 10,000 for its equipment. Their operating and maintenance costs are about Rs. 2500 per year for the first two years and then go up by approximately Rs. 1,500 per year. When such equipment replaced? The discount rate is 10% per year.

Solution:

Year n	M_t	Discount Factor d^{n-1}	Discounted Maint. Cost $M_t \cdot d^{n-1}$	Discounted Cumulative Maint. Cost $\sum M_t \cdot d^{n-1}$	TC $= \{(C - S) + \sum M_t\, d^{n-1}\}$	$\sum d^{n-1}$	Average Annual Cost $= \frac{TC}{\sum d^{n-1}}$

Ex. 6.6 A manufacturer is offered two machines A and B. A is priced at Rs. 10,000 and running costs are estimated as Rs. 1,600 for each of the first five years, increasing by Rs. 400 per year in the sixth and subsequent years. Machine B which has a same capacity as A cost Rs. 5,000 but will have a running coats of 2,400 per year for six year, increasing by Rs. 400 per year thereafter.

If money is worth 10% per year which machine should be purchased? (Assume that the machine will eventually be sold for scrap at a negligible price)

Solution:

Machine A

Year n	M_t	Discount Factor d^{n-1}	Discounted Maint. Cost $M_t \cdot d^{n-1}$	Discounted Cumulative Maint. Cost $\sum M_t \cdot d^{n-1}$	TC = $\{(C-S)+\sum M_t\, d^{n-1}\}$	$\sum d^{n-1}$	Average Annual Cost $= \frac{TC}{\sum d^{n-1}}$

Year n	M_t	Discount Factor d^{n-1}	Discounted Maint. Cost $M_t \cdot d^{n-1}$	Discounted Cumulative Maint. Cost $\sum M_t \cdot d^{n-1}$	TC = $\{(C-S) + \sum M_t \, d^{n-1}\}$	$\sum d^{n-1}$	Average Annual Cost $= \frac{TC}{\sum d^{n-1}}$

Ex. 6.7 A machine which requires an initial investment of Rs. 12,000 has its salvage value at the end of the year as Rs. $\left[7000-500(\boldsymbol{i}-1)\right]$. The operating and maintenance costs are given below:

Year	1	2	3	4	5	6	7	8	9
Maintenance Cost in Rs.	1100	1300	1700	2100	2300	2700	3100	3500	3900

Determine optimal replacement year when money increased by 12% ever year.

Solution:

n	d^{n-1}	S	$S \cdot d^n$	M_t	$M_t \cdot d^{n-1}$	$\sum(M_t \cdot d^{n-1})$	TC = $[\{C-(S \cdot d^n)\} + \sum M_t\, d^{n-1}]$	$\sum d^{n-1}$	Avg. annual Cost $= \frac{TC}{\sum d^{n-1}}$

Ex. 6.8 The following mortality rates have been observation for certain type of light bulbs.

Month	1	2	3	4	5
Percent failing by month end	10	25	50	80	100

There are 1000 bulbs in use and it costs Rs 10 to replace an individual bulb which has burnt out. If all bulbs were replaced simultaneously, it would cost Rs 2.5 per bulbs. It is proposed to replace all the bulbs at fixed interval, and individually those which fail between the intervals. What would be the best policy to adopt?

Solution:

Month i	Cumulative % failure up to the end of month	% failure during the month	Probability P_i that a new bulb shall fail during the month

Month i	Bulbs failing during i^{th} month	Bulbs replaced until i^{th} month	Cost of Individual Replacement TCI	Cost of Group Replacement TCG	Total Cost TC=TCI+TCG	Average Cost per month ATC = $\frac{TC}{n}$

6.7 Lab Examples

Ex. 6.9 The maintenance cost and resale price of machine M whose purchase price is Rs. 12,000 are given as:

Year	1	2	3	4	5	6	7
Maintenance Cost in Rs.	2600	3000	3400	4000	4700	5600	6600
Resale value in Rs.	7000	4500	3250	2600	2400	2400	2400

(a) Suggest the optimal period for the replacement of the machine.
(b) When this machine is two year old, another machine N, which is a new model of machine is available. The optimal period for replacement of this machine N is 4 year, with an avera cost of Rs. 4700. Should we change machine M with N? If so, when?

Solution:

Year n	Resale Price S	Maintenance Cost M_t	Cumulative Maintenance Cost $\sum M_t$	Total Cost $TC = C - S + \sum M_t$	Average Annual Cost $ATC = \frac{TC}{n}$

Ex. 6.10 For a machine, the following data are available

Year	0	1	2	3	4	5	6
Cost of spare in Rs.	-	200	400	700	1000	1400	1600
Salary of maintenance staff in Rs.	-	1200	1200	1400	1600	2000	2600
Loss due to breakdown in Rs.	-	600	800	700	1000	1200	1600
Resale value in Rs.	12000	6000	3000	1500	800	400	400

Determine the optimum period for the replacement of the above machine.

Solution:

Year n	Resale Price S	(Spare + Maintenance + Breakdown) M_t	Cumulative Maintenance Cost $\sum M_t$	Total Cost $TC = C - S + \sum M_t$	Average Annual Cost $ATC = \frac{TC}{n}$

Ex. 6.11 A piece of equipment costs Rs. 7,500 initially and requires Rs. 400 to be spent on its maintenance in the first year. The maintenance cost would increases by Rs. 500 per year in each of the subsequent years. Determine the optimal replacement for the machine when (i) future costs are not discounted, and (ii) future costs are discounted at the rate of 10% p.a.

Solution:

i) Without Discounted Future Cost

Year n	Maintenance Cost M_t	Cumulative Maintenance Cost $\sum M_t$	Total Cost $TC = C - S + \sum M_t$	Average Annual Cost $ATC = \frac{TC}{n}$

ii) With Discounted Future Cost

Year n	M_t	Discount Factor d^{n-1}	Discounted Maint. Cost $M_t \cdot d^{n-1}$	Discounted Cumulative Maint. Cost $\sum M_t \cdot d^{n-1}$	Total Cost TC $= \{(C - S) + \sum M_t \cdot d^{n-1}\}$	Cumulative Discount Factor $\sum d^{n-1}$	Average Annual Cost $= \frac{TC}{\sum d^{n-1}}$

Ex. 6.12 A manufacturer has to decide between two machines M_1 and M_2, about which pertinent information is given below:

	Machine M_1	Machine M_2
Cost	Rs. 5000	Rs. 2500
Maintenance cost	Rs. 800 p.a. for years 1,2,…,5, increasing by Rs. 200 every year thereafter	Rs. 1,200 p.a. for years 1,2,…,6, increasing by Rs. 200 every year thereafter
Scrap value	Nil	Nil
Cost of capital	(To be used as discounted rate) 10% p.a.	

Determine optimal replacement period of M_1 and M_2. Which of the two is a better alternative?

Solution:

Machine M_1

Year n	M_t	Discount Factor d^{n-1}	Discounted Maint. Cost $M_t \cdot d^{n-1}$	Discounted Cumulative Maint. Cost $\sum M_t \cdot d^{n-1}$	Total Cost TC $= \{(C - S) + \sum M_t\, d^{n-1}\}$	Cumulative Discount Factor $\sum d^{n-1}$	Average Annual Cost $= \frac{TC}{\sum d^{n-1}}$

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Machine M_2

Year n	M_t	Discount Factor d^{n-1}	Discounted Maint. Cost $M_t \cdot d^{n-1}$	Discounted Cumulative Maint. Cost $\sum M_t \cdot d^{n-1}$	Total Cost TC $= \{(C-S) + \sum M_t d^{n-1}\}$	Cumulative Discount Factor $\sum d^{n-1}$	Average Annual Cost $= \frac{TC}{\sum d^{n-1}}$

Ex. 6.13 Good lite Company has installed 2,000 electric bulbs of a certain brand. The company follows the policy of replacing the bulbs as and when they fail. Each replacement cost Rs. 2. The probability distribution of the life of the bulbs is as given here:

Life of bulb (weeks)	1	2	3	4	5
% of bulb	0.10	0.30	0.45	0.10	0.05

Determine the cost/ week of the replacement policy in long run.

Solution:

Ex. 6.14 A large computer installation contains 2000 components of identical nature which are subject to failure as per probability distribution that follows:

Month End	1	2	3	4	5
% Failure Data	10	25	50	80	100

Component which fail have to be replaced for efficient function of the system. If they are replaced as and when failure occurs, the cost of replacement per unit is Rs. 45. Alternatively, if all components are replaced in one lot at periodic intervals and individually replace only such failure occur between group replacements the cost of component replaced is Rs. 15.

Assess which policy of replacement would be economical.

Solution:

Month i	Cumulative % failure up to the end of month	% failure during the month	Probability P_i that a new component shall fail during the month

Month i	Components failing during i^{th} month	Components replaced until i^{th} month	Cost of Individual Replacement TCI	Cost of Group Replacement TCG	Total Cost TC=TCI+TCG	Average Cost per month ATC = $\frac{TC}{n}$