8

Decision Theory

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8.1 Introduction

- Everyday we make decisions and occasionally we make an important decision that can have immediate and/or long-term effects on our lives. Decisions like where to attend school, whether to rent or buy, whether your company should accept a merger proposal, whether to run a cool drink stall or a snacks stall and so on, are important decision—for which we would prefer to make the correct choice. A decision, in general, may be defined as the selection by the decision-maker of an act, considered to be best according to some predestinated standard, from among the several options.

8.1.1 Steps in Decision Theory Approach

- **Step 1:** Determine the various alternative courses of action (or choices or strategies) from which the final decision is to be made.
- **Step 2:** Identify the possible outcomes, called the states of nature or events for the decision problem. The events are beyond the control of the decision-maker.
- **Step 3:** Construct a pay off table
  The decision-maker now constructs a pay off table (table representing profit, benefit and so on) for each possible combination of alternative course of action and state of nature. If there are m alternative courses of action $A_1, \ldots, A_m$ and n states of nature $E_1, \ldots, E_n$, then the pay off matrix will be represented as
- **Step 4:** The decision-maker will choose the criterion which results in largest pay off. The criterion may be economic, quantitative or qualitative (for example, market share, profit, fragrance of a perfume and so on)

8.1.2 Decision-making Environments

- Decision analysis is used to determine optimum strategies where a decision-maker is faced with several decision alternatives. There are four types of decision-making environment:
  1. **Decisions under certainty**
     Whenever there exists only one outcome for a decision, we are dealing with this category. For example, the decision to purchase either a N.S.C. (National Saving Certificate), Indira Vikas Patra, or deposit in N.S.S. (National Saving Scheme) is one in which it is reasonable to assume the complete information about the further because there is no doubt that the Indian Government will pay the interest as it falls due and the principal at maturity.
     As other examples we can take linear programming transportation problem, assignment problem and sequencing and so on.
  2. **Decisions under uncertainty**
     These refer to situations where more than one outcome can result from any single decision. That is, here more-than one states of nature exist but the decision-maker lacks sufficient knowledge to allow him to assign probabilities to the various states of nature. For example, the probability that Mr X will be the Prime Minister of the country 15 years from now is not known.
  3. **Decisions under risk**
     These refer to decision situations wherein the decision-maker chooses from among several possible outcomes where the probability of occurrence can be stated objectively from the past data.
  4. **Decisions under conflict**
     In many situations, neither states of nature are completely known nor are they completely uncertain. Partial knowledge is available and therefore, it may be termed as decision-making under 'partial uncertainty'.

As an example, we can consider the situation of conflict involving two or more competitors marketing the same product.

8.2 Decision Under Uncertainty

- Under conditions of uncertainty, the decision-maker has a knowledge about the states of nature that happen but lacks the knowledge about the probabilities of their occurrence. Situations like launching a new product fall under this category. The insufficient data lead to a more complex decision model and perhaps, a less satisfactory solution. However, one uses scientific methods to exploit the available data to the fullest extent.
- In decisions under uncertainty, situations exist in which two (or more) opponents with conflicting objectives try to make decisions with each trying to gain at the cost of the other(s). These situations are different since the decision-maker is working against an intelligent opponent. The theory governing these types of decision problems is called the theory of games and will be taken up later in this chapter.
- Different persons have suggested several decision rules for making a decision under such situations.

8.2.1 Criterion of Pessimism (Minimax or Maximin)

- The Maximin criterion is based upon the 'Conservative approach' to assume that the worst possible is going to happen. The decision-maker considers each strategy and locates the minimum pay off for each, and then select that alternative which maximises the minimum pay off.
- Thus, this criterion involves two steps.
  - Step 1: Find the minimum assured pay off for each alternative (course of action).
  - Step 2: Choose that alternative which corresponds to the maximum of the above minimum pay off.
- When dealing with costs, the maximum cost associated with each alternative is considered and the alternative that minimises this maximum cost is chosen. This is known as Minimax criterion and it involves two steps.
  - Step 1: Determine the maximum possible cost for each alternative.
  - Step 2: Choose that alternative which corresponds to the minimum of the above costs.

8.2.2 Criterion of Optimism (Maximax or Minimin)

- This criterion is based upon extreme optimism.
- In this criterion the decision-maker ensures that be should not miss the opportunity to achieve the greatest possible pay off or lowest possible cost. In maximax criterion the decision-maker selects that particular strategy which corresponds to the maximum pay off for each strategy.
- Thus, the maximax criterion consists of the following two steps.
  - Step 1: Determine the maximum possible pay off for each alternative
  - Step 2: Select that alternative which corresponds to the maximum of the above maximum pay offs.
- In decision problems dealing with costs, the minimum for each alternative is considered and then the alternative which minimises the above minimum cost is selected. This is termed as minimin principle.

8.2.3 Laplace Criterion or Equally Likely Decision Criterion

- This criterion is based upon what is known as the principal of insufficient reason.
- Since the probabilities associated with the occurrence of various events are unknown, there is not enough information to conclude that these probabilities will be different. Hence, it is assumed that all states of nature will occur with equal probability. That is, each state of nature is assigned an equal

probability. As states of nature are mutually exclusive and collectively exhaustive, so the probability of each of these much be 1/(number of states of nature).

- The working procedure is as follows.
  - Step 1: Assign equal probabilities 1/(number of states of nature) to each pay off of a strategy.
  - Step 2: Determine the expected pay off value for each alternative.
  - Step 3: Select that alternative which corresponds to the maximum (and minimum for cost) of the above expected pay offs.

8.2.4 Criterion of Realism (Hurwicz Criterion)

- This criterion suggests that a rational decision-maker should be neither be completely optimistic nor pessimistic and therefore, must display a mixture of both. Hurwicz, who suggestes this criterion, introduced the idea of a coefficient of optimism (denoted by α) to measure the decision-maker's degree of optimism. This coefficient lies between 0 and 1, where 0 represents a completely pessimistic attitude about the future and 1 a completely optimistic attitude about the future. Thus, if α is the coefficient of optimism, then (1 – α) will represents the coefficient of pessimism.
- The working procedure is summarised as follows:
  1. Decide the coefficient of optimism a and then the coefficient of pessimism 1 – a.
  2. Determine the maximum as well as minimum pay off for each alternative and obtain the quantities $h = \alpha \times maximum\ for\ each\ alternative + (1-\alpha) \times minimum\ for\ each\ alternative.$
  3. Select an alternative with value of h as maximum.

8.2.5 Criterion of Regret (Savage Criterion) or Minimax Regret Criterion

- This decision criterion was developed by L.J. Savage. He pointed out that the decision-maker might experience regret after the decision has been made and the states of nature i.e., events have occurred.Thus the decision-maker should attempt to minimize regret before actually selecting a particular alternative (strategy).
- The basic steps involved in this criterion are:
  1. Determine the amount of regret corresponding to each event for every alternative. The regret for *j*th event corresponding to *i*th alternative is given by
     *i*th regret = (maximum payoff - *i*th payoff) for the *j*th event.
  2. Determine the maximum regret amount for each alternative.
  3. Choose the alternative which corresponds to the minimum of the above maximum regrets.

8.3 Decision Under Certainty

- Since under this environment, only one state of nature exists, the decision-maker simply picks up the best pay off in the one column and chooses the associated alternative. Under conditions of certainty, the particular state of nature is associated with probability 1. Though the state of nature is only one, possible alternatives could be numerous.
- Linear programming, transportation and assignment techniques, input out analysis, activity analysis and economic order quantity models are used for such situations. Few complex managerial decision making problems, however, ever enjoy the luxury of having complete information about the future and thus decision-making under certainty is of little consequential interests.

8.4 Decision-Making Under Risk

- Most business decisions may have to be made under conditions of risk. Here, more than one states of nature exist and the decision-maker has sufficient information to assign probabilities to each of these states. These probabilities could be obtained form the past records or from simply the subjective judgment of the decision-maker. Under conditions of risk, a number of decision criteria are available which could be of help to the decision-maker. The most popular criterion for evaluating the alternatives is the expected monetary value/expected opportunity loss of the expected pay off.

8.4.1 Expected Value Criterion

- This criterion requires the calculation of the expected value of each decision alternative which is the sum of the weighted payoffs for that alternative, where the weights are the probabilities assigned to the states of nature that can happen. Also known as expected monetary value (EMV) criterion, it consists of the following steps :
  1. Construct a conditional pay-off table listing the alternative decisions and the various states of nature. Enter the conditional profit for each decision-event combination along with the associated probabilities.
  2. Calculate the EMV for each decision alternative by multiplying the conditional profits by assigned probabilities and adding the resulting conditional values.
  3. Select the alternative that yields the highest EMV.

8.4.2 Expected Opportunity Loss (EOL) Criterion

- An alternative approach (to maximizing EMV approach) is to minimize expected opportunity loss (EOL). Expected opportunity loss (or expected value of regrets) represents the amount by which maximum possible profit will be reduced under various possible stock actions. The course of action that minimizes these losses or reductions is the optimal decision alternative. The procedure to calculate expected opportunity losses is as follows:
  1. Prepare the conditional profit table for each decision-event combination and write the associated probabilities.
  2. For each event, determine the conditional opportunity loss (COL) by subtracting the payoff from the maximum payoff for that event.
  3. Calculate the expected opportunity loss (EOL) for each decision alternative by multiplying the COL's by the associated probabilities and then adding the values.
  4. Select the alternative that yields the lowest EOL.

8.5 Decision Trees

- A situations may arise when decision-maker needs to revise his previous decisions on getting new information and make a sequence of other decisions. Thus, the problem becomes a multi-stage decision problem because the consequence of one decision affects future decisions. For example, in the process of marketing a new product, the first decision is often test marketing and the alternative courses of action might be either intensive testing or gradual testing. Given various possible consequences—favourable fair, or poor, the decision-maker may be required to decide between redesigning the product, and aggressive advertising campaign or complete withdrawal of product and so on. Given that decision, there will be an outcome which leads to another decision and so on.
- **Decision Tree** is a graphical representation of various decision alternatives and the sequence of events as if they were branches of a tree.
- A decision node is usually represented by a square and indicates places where a decision-maker must make a decision. Each branch leading away from a decision node represents one of the several possible courses of action available to the decision-maker. The chance node is represented by a circle and indicates a point at which the decision-maker will discover the response to his decision, that is, different possible outcomes which can result from a chosen course of action. The respective pay offs

and the probabilities associated with alternative courses and the chance events are shown alongside these branches. At the terminal of the chance branch are shown the expected values of the outcome are shown.

- The general approach used in decision tree analysis is to work back ward through the tree from right to left, computing the expected value of each chance node. We then choose the particular branch leaving a decision node which leads to the chance node with the highest expected value. This is known as roll back or fold back process.

8.5.1 Steps in Decision Tree Analysis

1. Identify the decision points and the alternative courses of action at each decision point systematically.
2. At each decision point determine the probability and the pay off associated with each course of action.
3. Commencing from the extreme right end, compute the expected pay offs (EMV) for each course of action.
4. Choose the course of action that yields the best pay off for each decisions.
5. Proceed backwards to the next stage of decision points.
6. Repeat above steps till the first decision point is reached.
7. Finally, identify the course of action to be adopted from the beginning to the end under different possible outcomes for the situation as a whole.

8.5.2 Advantages of Decision Tree Approach

- It structures the decision process and helps decision-making in an orderly, systematic and sequential manner.
- It requires the decision-maker to examine all possible outcomes, whether desirable or undesirable.
- It communicates the decision-making process to other in an easy and clear manner, illustrating each assumption about the future.
- It displays the logical relationship between the parts of a complex decision and identifies the time sequence in which various actions and subsequent events would occur.
- It is especially useful in situations wherein the initial decision and its outcome affects the subsequent decisions. It can be applied in various fields such as introduction of a new product, marketing, make or buy decisions, investment decisions and so on.

8.5.3 Limitations of Decision Tree Approach

- Decision tree diagrams become more complicated as the number of decision alternative increases and more variable are introduced.
- It becomes highly complicated when inter-dependent alternatives and dependent variables are present in the problem.
- It assumes that utility of money is linear with money.
- It analyses the problem in terms of expected values and thus yields an ‘average’ valued solution.
- There is often inconsistency in assigning probabilities for different events.