3

Velocity and Acceleration Analysis

Contents

3.1 Introduction

Linear Velocity: It may be defined as the rate of change of linear displacement of a body with respect to the time. If the displacement is along a circular path, then the direction of linear velocity at any instant is along the tangent at that point.

Linear Acceleration: It may be defined as the rate of change of linear velocity of a body with respect to the time. The negative acceleration is also known as deceleration or retardation.

There are many methods for determining the velocity of any point on a link in a mechanism whose direction of motion (i.e. path) and velocity of some other point on the same link is known in magnitude and direction, yet the following two methods:

- Instantaneous centre method
- Relative velocity method

The instantaneous centre method is convenient and easy to apply in simple mechanisms, whereas the relative velocity method may be used to any configuration diagram.

3.2 The Motion of a Link

Consider two points A and B on a rigid link AB, as shown in *Fig.3.1* (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.
Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.
The relative velocity of B with respect to A (i.e. V_{BA}) is represented by the vector *ab* and is perpendicular to the line AB as shown in *Fig.3.1* (b).

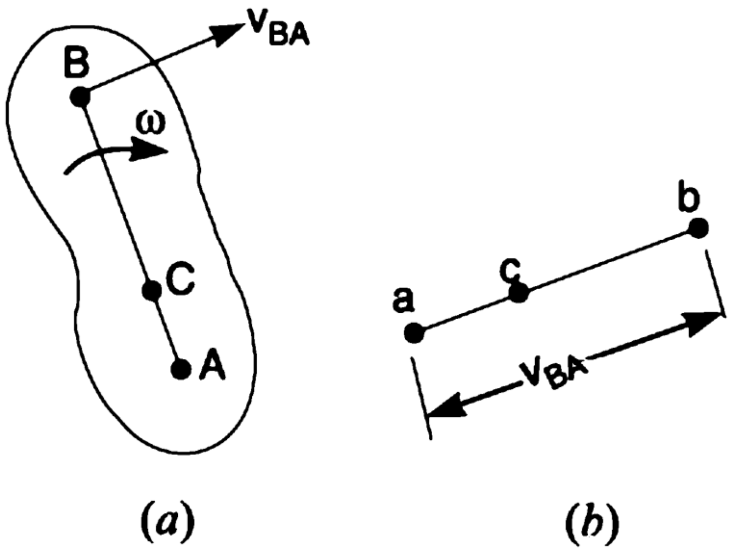


Fig.3.1 - Motion of a Link

The velocity of point B with respect to A

$$v_{BA} = \omega \times AB \qquad \text{Eq. (3.1)}$$

The velocity of point C on AB with respect to A

$$v_{CA} = \omega \times AC \qquad \text{Eq. (3.2)}$$

From *Eq. (3.1)* and *Eq. (3.5)*,

$$\frac{v_{CA}}{v_{BA}} = \frac{\omega \times AC}{\omega \times AB} = \frac{AC}{AB} \qquad \textit{Eq. (3.3)}$$

Thus, we see from *Eq. (3.5)*, that the point c on the vector ab divides it in the same ratio as C divides the link AB.

3.2.1 The Velocity of a Point on a Link by Relative Velocity Method

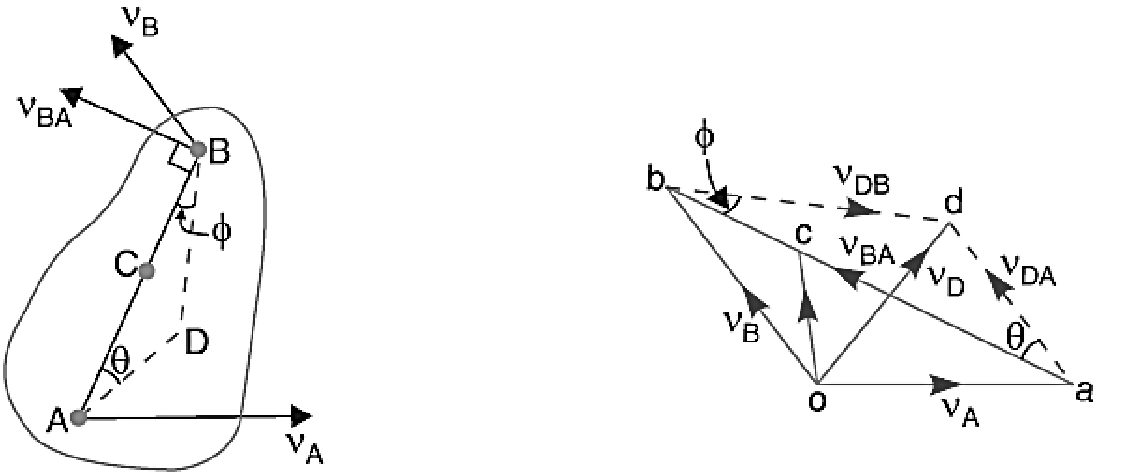


Fig.3.2 – Motion of points on a link and its velocity diagram

Consider two points A and B on a link AB. Let the absolute velocity of the point A i.e. V_A is known in magnitude and direction and the absolute velocity of the point B i.e. v_B is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown in *Fig.3.2*. The velocity diagram is drawn as follows:

- Take some convenient point o, known as the pole.
- Through o, draw oa parallel and equal to V_A, to some suitable scale.
- Through a, draw a line perpendicular to AB. This line will represent the velocity of B with respect to A, i.e. V_{BA}.
- Through o, draw a line parallel to v_B intersecting the line of V_{BA} at b.
- Measure ob, which gives the required velocity of point B (v_B), to the scale.

3.3 Velocities in Slider Crank Mechanism

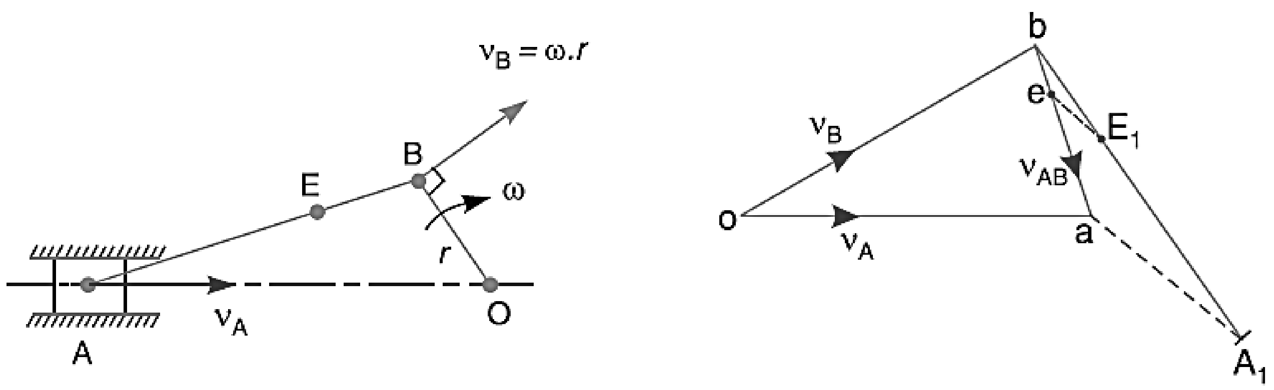


Fig.3.3 – Slider crank mechanism and its velocity diagram

A slider-crank mechanism is shown in *Fig.3.3*. The slider A is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity ω rad/s. Therefore, the velocity of B i.e. v_B is known in magnitude and direction. The slider reciprocates along the line of stroke AO.
The velocity of the slider A (i.e. V_A) may be determined by the relative velocity method as discussed below:

- From any point o, draw vector ob parallel to the direction of v_B (or perpendicular to OB) such that ob = v_B = ω.r, to some suitable scale.
- Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB. Now draw vector ba perpendicular to AB to represent the velocity of A with respect to B i.e. v_{AB}.
- From point o, draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors ba and oa intersect at a. Now oa represents the velocity of the slider I.e. V_A, to the scale.

The angular velocity of the connecting rod A B (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$

3.3.1 Rubbing Velocity at a Pin Joint

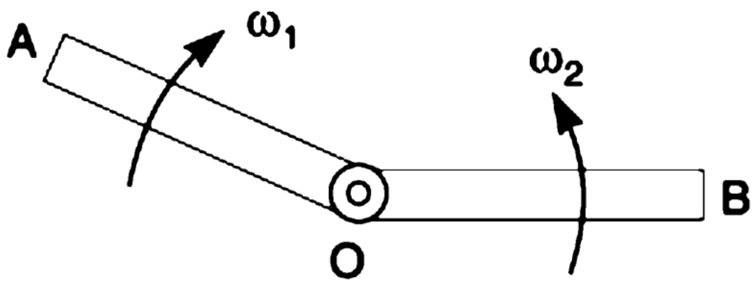


Fig.3.4 - Links connected by pin joints

The links in a mechanism are mostly connected by means of pin joints. The ***rubbing velocity*** is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.
Consider two links OA and OB connected by a pin joint at O as shown in *Fig.3.4*.
Let,

ω_1 = angular velocity of link OA
ω_2 = angular velocity of link OB

According to the definition, rubbing velocity at the pin joint O,

$$= (\omega_1 - \omega_1) \times r \text{ if the links move in the same direction}$$

$$= (\omega_1 + \omega_1) \times r \text{ if the links move in the same direction}$$

3.4 The Velocity of a Point on a Link by Instantaneous Centre Method

The instantaneous centre method of analyzing the motion in a mechanism is based upon the concept that any displacement of a body (or a rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centre or virtual centre of rotation.

Prof. Sunil G. Janiyani, Department of Mechanical Engineering

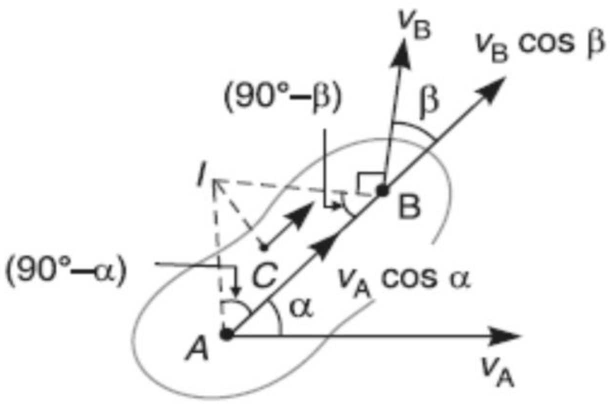


Fig.3.5 - Velocity of a point on a link

The velocities of points A and B, whose directions are given a link by angles α and β as shown in *Fig.3.5*. If vA is known in magnitude and direction and vB in direction only, then the magnitude of vB may be determined by the instantaneous centre method as discussed below:

- Draw AI and BI perpendiculars to the directions V_A and v_B respectively. Let these lines intersect at I, which is known as instantaneous centre or virtual centre of the link. The complete rigid link is to rotate or turn about the centre I.
- Since A and B are the points on a rigid link, therefore there cannot be any relative motion between them along line AB.
- Now resolving the velocities along AB,

$$v_A \times \cos\alpha = v_B \times \cos\beta$$

$$\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90-\beta)}{\sin(90-\alpha)} \qquad \text{Eq. (3.4)}$$

- Applying Lami's theorem to triangle ABI,

$$\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}$$

$$\therefore \frac{AI}{BI} = \frac{\sin(90-\beta)}{\sin(90-\alpha)} \qquad \text{Eq. (3.5)}$$

- From *Eq. (3.4)* and *Eq. (3.5)*,

$$\frac{v_A}{v_B} = \frac{AI}{BI}$$

$$\therefore \frac{v_A}{AI} = \frac{v_B}{BI} = \omega$$

- If C is any other point on a link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI}$$

3.5 Properties of Instantaneous Centre

The following properties of the instantaneous centre are important:

- A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.

- The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (i.e. instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link. In other words, the velocity of the instantaneous centre relative to any third rigid link will be same whether the instantaneous centre is regarded as a point on the first rigid link or on the second rigid link.

3.5.1 Number of Instantaneous Centre in a Mechanism

The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number of instantaneous centres is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centres,

$$N = \frac{n\,(n-1)}{2}, where\ n = Number\ of\ Link$$

3.5.2 Types of Instantaneous Centres

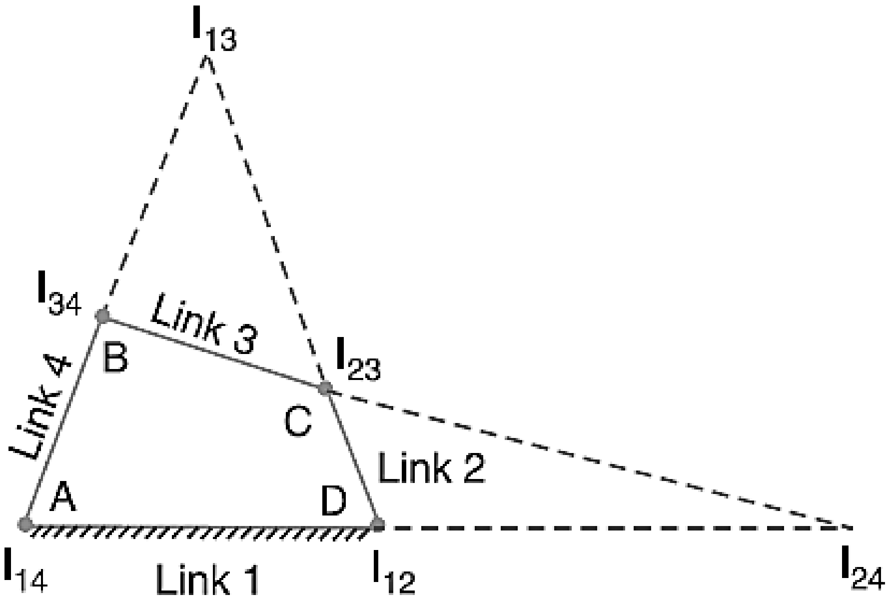


Fig.3.6 - Types of instantaneous centres

The instantaneous centres for a mechanism are of the following three types:

1. Fixed instantaneous centres,
2. Permanent instantaneous centres, and
3. Neither fixed nor permanent instantaneous centres.

The first two types i.e. fixed and permanent instantaneous centres are together known as primary instantaneous centres and the third type is known as secondary instantaneous centres.

Consider a four-bar mechanism ABCD as shown in *Fig.3.6*.

The instantaneous centres I_{12} and I_{14} are called the ***fixed instantaneous centres*** as they remain in the same place for all configurations of the mechanism.

The instantaneous centres I_{23} and I_{34} are the ***permanent instantaneous centres*** as they move when the mechanism moves, but the joints are of permanent nature.

The instantaneous centres I_{13} and I_{24} are ***neither fixed nor permanent instantaneous centres*** as they vary with the configuration of the mechanism.

3.5.3 Location of Instantaneous Centres

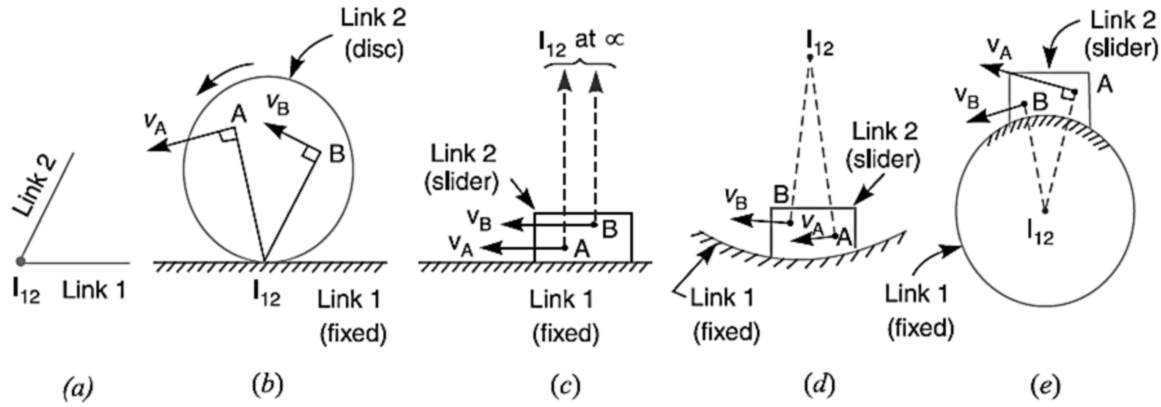


Fig.3.7 - Location of instantaneous centres

The following rules may be used in locating the instantaneous centres in a mechanism:

1. When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies in the centre of the pin as shown in *Fig.3.7 (a)*. Such an instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.

2. When the two links have a pure rolling contact (i.e. link 2 rolls without slipping upon the fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in *Fig.3.7 (b)*. The velocity of any point A on the link 2 relatives to fixed link 1 will be perpendicular to $I_{12}A$ and is proportional to $I_{12}A$.

3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases:

(a) When the link 2 (slider) moves on fixed link 1 having a straight surface as shown in *Fig.3.7 (c)*, the instantaneous centre lies at infinity and each point on the slider have the same velocity.

(b) When the link 2 (slider) moves on fixed link 1 having a curved surface as shown in *Fig.3.7 (d)*, the instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

(c) When the link 2 (slider) moves on fixed link 1 having a constant radius of curvature as shown in *Fig.3.7 (e)*, the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.

3.6 Kennedy's Theorem

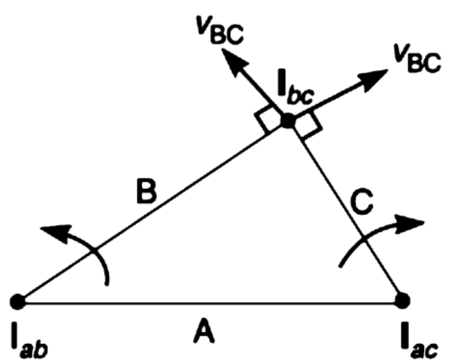


Fig.3.8 - Aronhold Kennedy's theorem

Aronhold Kennedy's theorem states that "if three bodies move relative to each other, they have three instantaneous centres and lie on a straight line."
Consider three kinematic links A, B and C having relative plane motion. The number of instantaneous centres (N) is given by

$$N = \frac{n\,(n-1)}{2} = \frac{3\,(3-1)}{2} = 3$$

The two instantaneous centres at the pin joints of B with A and C with A (i.e. I_{ab} and I_{ac}) are the permanent instantaneous centre. According to Aronhold Kennedy's theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}.
In order to prove this, let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in *Fig.3.8*. The point I_{bc} belongs to both the links B and C.
Let us consider the point I_{bc} on the link B. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc}. Now consider the point I_{bc} on the link C. Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc}.

3.7 Acceleration Diagram for a Link

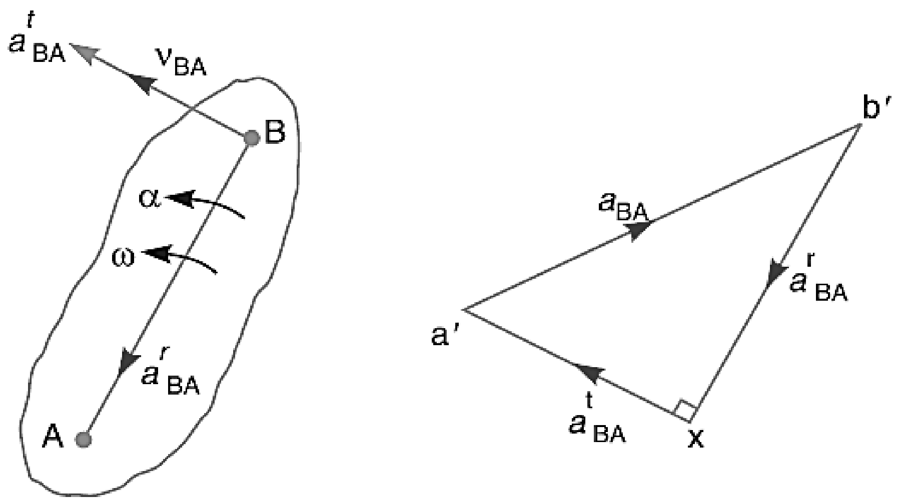


Fig.3.9 - Acceleration of a link

Consider two points A and B on a rigid link. Let the point B moves with respect to A, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link AB.
We know that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components.

- The ***centripetal or radial component***, which is perpendicular to the velocity of the particle at the given instant.
- The ***tangential component***, which is parallel to the velocity of the particle at the given instant.

Thus for a link A B, the velocity of point B with respect to A (i.e. V_{BA}) is perpendicular to the link AB. Since point B moves with respect to A with an angular velocity of ω rad/s, therefore a centripetal or radial component of the acceleration of B with respect to A

$$a_{BA}^{r} = \omega^2 \times Length\ of\ link\ AB = \omega^2 \times AB = {v_{BA}^2}/{AB}$$

This radial component of acceleration acts perpendicular to the velocity V_{BA}. In other words, it acts parallel to the link AB. The tangential component of the acceleration of B with respect to A,

$$a_{BA}^{t} = \alpha \times Length\ of\ link\ AB = \alpha \times AB$$

This tangential component of acceleration acts parallel to the velocity V_{BA}. In other words, it acts perpendicular to the link AB.

In order to draw the acceleration diagram for a link AB, as shown in *Fig.3.9*, from any point b', draw vector b'x parallel to BA to represent the radial component of acceleration of B with respect to A.
From point x, draw vector xa' perpendicular to BA to represent the tangential component of acceleration of B with respect to A.
Join b'a'. The vector b'a' (known as acceleration image of the link AB) represents the total acceleration of B with respect to A and it is the vector sum of radial component and the tangential component of acceleration.

3.7.1 Acceleration of a Point on a Link

Consider two points A and B on the rigid link, as shown in *Fig.3.10*. Let the acceleration of the point A i.e. a_A is known in magnitude and direction and the direction of the path of B is given.
The acceleration of point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below:

- From any point o', draw vector o'a' parallel to the direction of absolute acceleration at point A i.e. a_A, to some suitable scale.

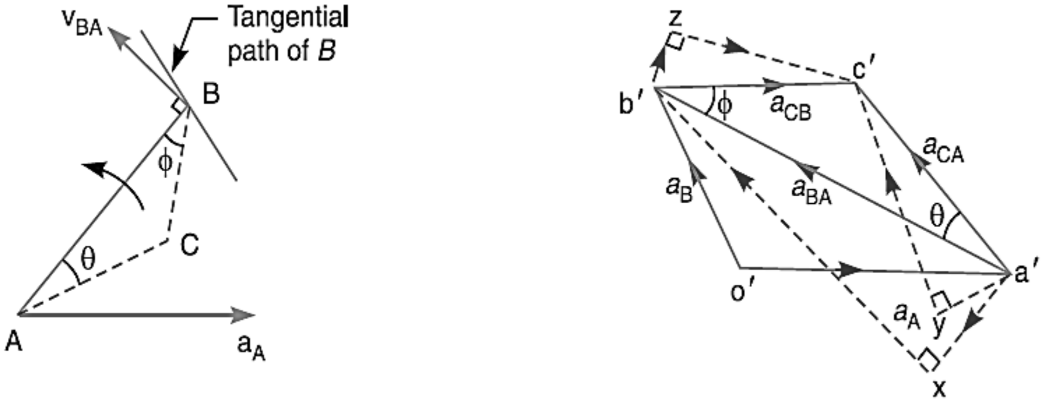


Fig.3.10 - Acceleration of a point on a link

- The acceleration of B with respect to A i.e. a_{BA} has the following two components:
  - The radial component of the acceleration of B with respect to A i.e. $\boldsymbol{a_{BA}^r}$
  - The tangential component of the acceleration B with respect to A i.e. $\boldsymbol{a_{BA}^t}$
- Draw vector a’x parallel to the link AB such that,

$$vector\ a'x = a_{BA}^r = v_{BA}^2/AB$$

- From point x, draw vector xb’ perpendicular to AB or vector a’x and through o’ draw a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B.
- By joining the points a' and b' we may determine the total acceleration of B with respect to A i.e. a_{BA}. The vector a' b' is known as acceleration image of the link AB.
- For any other point C on the link, draw triangle a' b' c' similar to triangle ABC. Now vector b' c' represents the acceleration of C with respect to B i.e. a_{CB}, and vector a' c' represents the acceleration of C with respect to A i.e. a_{CA}. As discussed above, a_{CB} and a_{CA} will each have two components as follows:
  a. a_{CB} has two components; $\boldsymbol{a_{CB}^r}$ and $\boldsymbol{a_{CB}^t}$ as shown by triangle b’zc’ in fig.b
  b. a_{CA} has two components; $\boldsymbol{a_{CA}^r}$ and $\boldsymbol{a_{CA}^t}$ as shown by triangle a’yc’
- The angular acceleration of the link AB is obtained by dividing the tangential component of acceleration of B with respect to A to the length of the link.

$$\alpha_{AB} = a_{BA}^t/AB$$

3.8 Acceleration in Slider Crank Mechanism

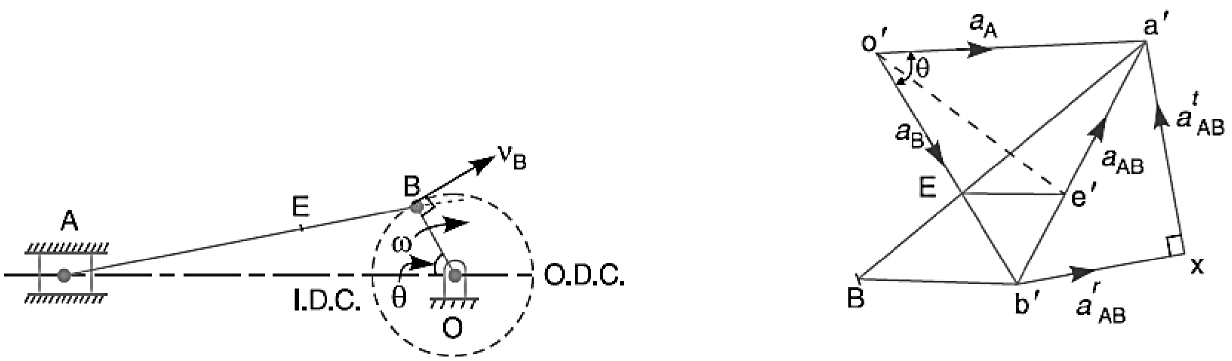


Fig.3.11 - Acceleration in the slider-crank mechanism and its acceleration diagram

A slider-crank mechanism is shown in *Fig.3.11*.

Let the crank OB makes an angle θ with the inner dead centre (I.D.C) and rotates in a clockwise direction about the fixed point O with uniform angular velocity ω_{BO} rad/s.

The velocity of B with respect to O or velocity of B (because O is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB \ acting \ tangentially \ at \ B$$

The centripetal or radial acceleration of B with respect to O or acceleration of B (Because O is a fixed point),

$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{BO}$$

The acceleration diagram may be drawn as discussed below:

a) Draw vector o' b' parallel to BO and set off equal in the magnitude of $a^r_{BO} = a_B$, to some suitable scale.

b) From point b', draw vector b'x parallel to BA. The vector b'x represents the radial component of the acceleration of A with respect to B whose magnitude is given by:

$$a^r_{AB} = v^2_{AB}/BA$$

Since point B moves with constant angular velocity, therefore there will be no tangential component of the acceleration.

c) From point x, draw vector xa' perpendicular to b'x (or AB). The vector xa' represents the tangential components of the acceleration of A with respect to B.

d) Since point A reciprocates along AO, therefore the acceleration must be parallel to velocity. Therefore from o', draw o' a' parallel to AO, intersecting the vector xa' at a'.

e) The vector b' a', which is the sum of the vectors b' x and x a', represents the total acceleration of A with respect to B i.e. a_{AB}. The vector b'a' represents the acceleration of the connecting rod AB.

f) The acceleration of any other point on A B such as E may be obtained by dividing the vector b' a' at e' in the same ratio as E divides A B. In other words,

$$a'e'/a'b' = AE/AB$$

g) The angular acceleration of the connecting rod A B may be obtained by dividing the tangential component of the acceleration of A with respect to B to the length of AB. In other words, the angular acceleration of AB,

$$\alpha_{AB} = a^t_{AB}/AB$$

3.9 Problems

Ex. 3.1 **In a four-bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.**

Solution: Given Data:

N_{BA} = 120 r.p.m.

$\therefore \omega_{BA}$ = 2 π × 120/60 = 12.568 rad/s

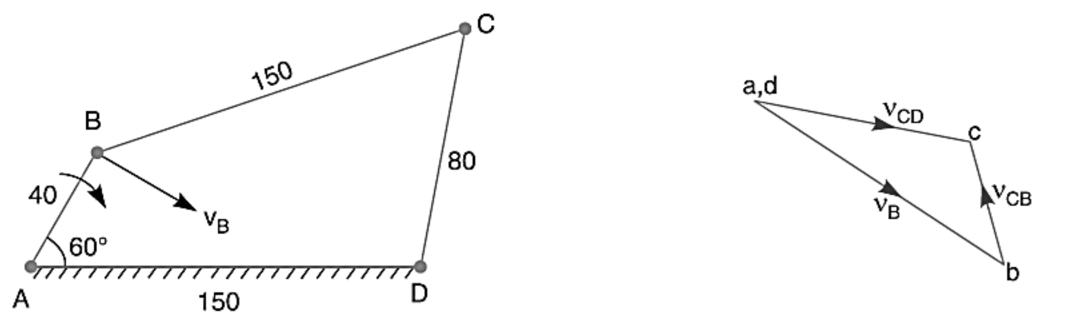
Since the length of crank AB = 40 mm = 0.04 m, therefore the velocity of B with respect to A or velocity of B, (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$

Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram.

Draw vector ab perpendicular to BA, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (i.e. v_{BA} or v_B) such that

$$\text{Vector } ab = v_{BA} = v_B = 0.503 \text{ m/s}$$



(a) Space diagram (All dimensions in mm). (b) Velocity diagram.

Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (i.e. v_{CB}) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c.

By measurement, we find that

$$V_{CD} = v_C = \text{vector } dc = 0.385 \text{ m/s}$$

Angular velocity of link CD,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \ rad/s$$

Ex. 3.2 **The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine: 1. The velocity of the piston, 2. Angular velocity of connecting rod, 3. The velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4. velocities of rubbing at the pins of the crankshaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. Position and linear velocity of any point G on the connecting rod which has the least velocity relative to the crankshaft.**

Solution: Given Data:

N_{BO} = 180 r.p.m.

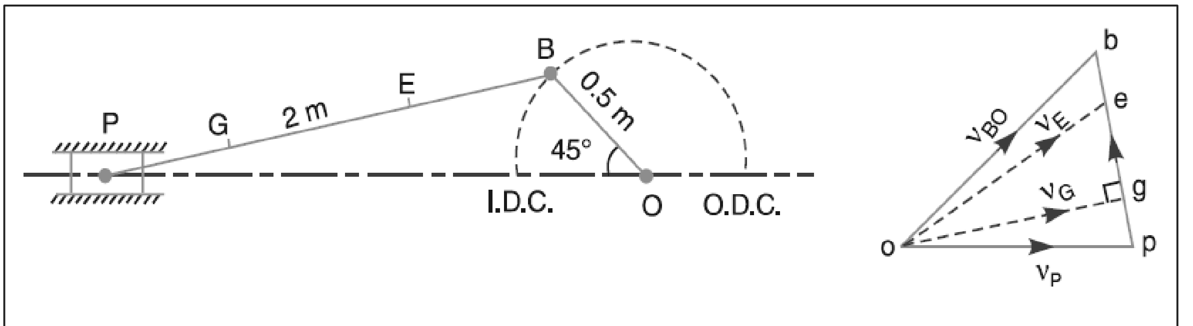
$\therefore \omega_{BO}$ = 2 π × 180/60 = 18.852 rad/s

The extreme positions of the crank are shown in Figure.

Since the crank length OB = 0.5 m, therefore the linear velocity of B with respect to O or velocity of B (because O is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

First of all, draw the space diagram and then draw the velocity diagram as shown in the figure.



(*a*) Space diagram. (*b*) Velocity diagram.

Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or velocity of B such that

$$\text{vector } ob = v_{BO} = v_B = 9.426 \text{ m/s}$$

From point b, draw vector bp perpendicular to BP to represent the velocity of P with respect to B (i.e. v_{PB}) and from point o, draw vector op parallel to PO to represent the velocity of P with respect to O (i.e. v_{PO} or simply v_P). The vectors bp and op intersect at point p.

By measurement, we find that velocity of piston P,

$$\boldsymbol{v_P = vector\ op = 8.15\ m/s}$$

From the velocity diagram, we find that the velocity of P with respect to B,

$$\boldsymbol{v_{PB} = \ vector\ bp \ = \ 6.8\ m/s}$$

Since the length of connecting rod PB is 2 m, therefore the angular velocity of the connecting rod,

$$\boldsymbol{\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4\ rad/s}$$

$$\boldsymbol{v_E = vector\ oe = 8.5\ m/s}$$

The velocity of rubbing at the pin of crank-shaft,

$$= \frac{d_0}{2} \times \omega_{BO} = 0.47\ m/s$$

The velocity of rubbing at the pin of the crank,

$$= \frac{d_B}{2} (\omega_{BO} + \omega_{PB}) = 0.6675\ m/s$$

The velocity of rubbing at the pin of the crank,

$$= \frac{d_c}{2} \times \omega_{PB} \ = 0.051\ m/s$$

By measurement, we find that,

$$vector\ bg = 5\ m/s$$

By measurement, we find the linear velocity of point G,

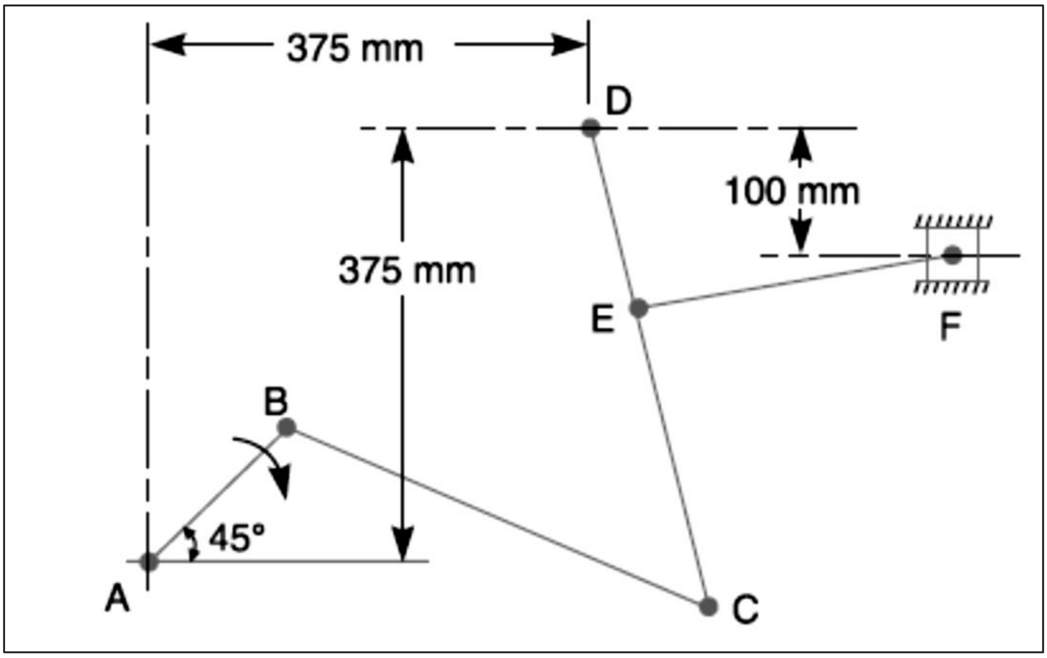
$$v_G = vector\ og = 8\ m/s$$

Ex. 3.3 **The mechanism, as shown in the figure, has the dimensions of various links as follows: AB = DE = 150 mm; BC = CD = 450 mm; EF = 375 mm. The crank AB makes an angle of 45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC. The block F moves in the horizontal guides, being driven by the link EF. Determine: 1. velocity of the block F, 2. angular velocity of DC, and 3. rubbing speed at the pin C which is 50 mm in diameter.**

Solution: Given Data:

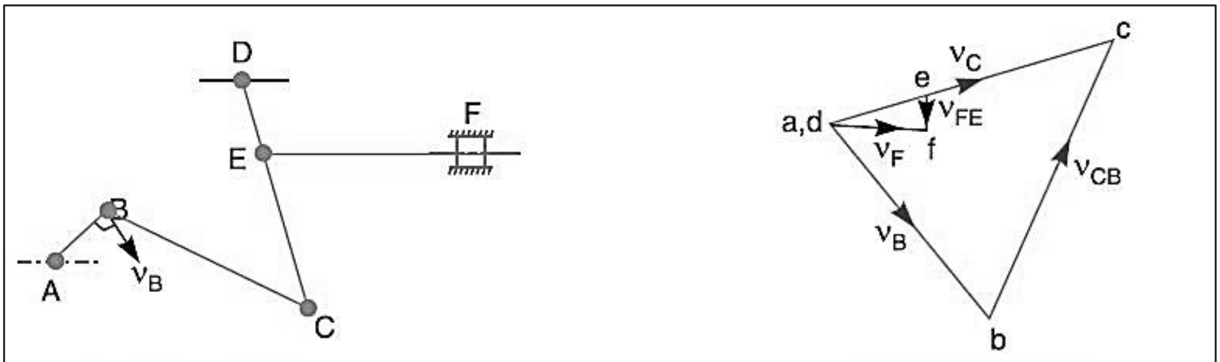
N_{BA} = 120 RPM

$\therefore \omega_{BA} = 2\,\pi \times 120/60 = 4\,\pi$ rad/s



Since the crank length AB = 150 mm = 0.15 m, therefore velocity of B with respect to A or simply velocity of B (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\,\pi \times 0.15 = 1.885 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram.

First of all, draw the space diagram, to some suitable scale, as shown in the figure. Now the velocity diagram is drawn as discussed below:

Since the points A and D are fixed, therefore these points are marked as one point as shown in the figure. Now from point a, draw vector ab perpendicular to A B, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B, such that

$$\text{Vector } ab = v_{BA} = v_B = 1.885 \text{ m/s}$$

The point C moves relative to B and D, therefore draw vector bc perpendicular to BC to represent the velocity of C with respect to B (i.e. v_{CB}), and from point d, draw vector dc perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c.
Since the point E lies on DC, therefore divide vector dc in e in the same ratio as E divides CD. In other words,

$$ce/cd = CE/CD$$

From point e, draw vector ef perpendicular to EF to represent the velocity of F with respect to E (i.e. v_{FE}) and from point d draw vector df parallel to the path of motion of F, which is horizontal, to represent the velocity of F i.e. v_F. The vectors ef and df intersect at f. By measurement, we find that velocity of the block F,

$$v_F = \text{vector } df = 0.7 \text{ m/s}$$

By measurement from velocity diagram, we find that velocity of C with respect to D,

$$v_{CD} = \text{vector } dc = 2.25 \text{ m/s}$$

Since the length of link DC = 450 mm = 0.45 m, therefore the angular velocity of DC,

$$\omega_{DC} = \frac{v_{CD}}{DC} = 5\frac{rad}{s}$$

From the velocity diagram, we find that velocity of C with respect to B,

$$v_{CB} = \text{vector } bc = 2.25 \text{ m/s}$$

Angular velocity of BC,

$$\omega_{CD} = \frac{v_{CD}}{BC} = \frac{2.25}{0.45} = 5\, rad/s$$

Rubbing speed at the pin C,

$$= (\omega_{CB} - \omega_{CD})\, r_C$$
$$= (5 - 5)\, 0.025$$
$$= 0$$

Ex. 3.4 The crank of the slider-crank mechanism rotates clockwise at a constant speed of 300 RPM. The crank is 150 mm and the connecting rod is 600 mm long. Determine:

1. **Linear velocity and acceleration of the midpoint of the connecting rod, and**
2. **Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from the inner dead centre position**

Solution: Given Data:

N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s;

OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713\, m/s$$

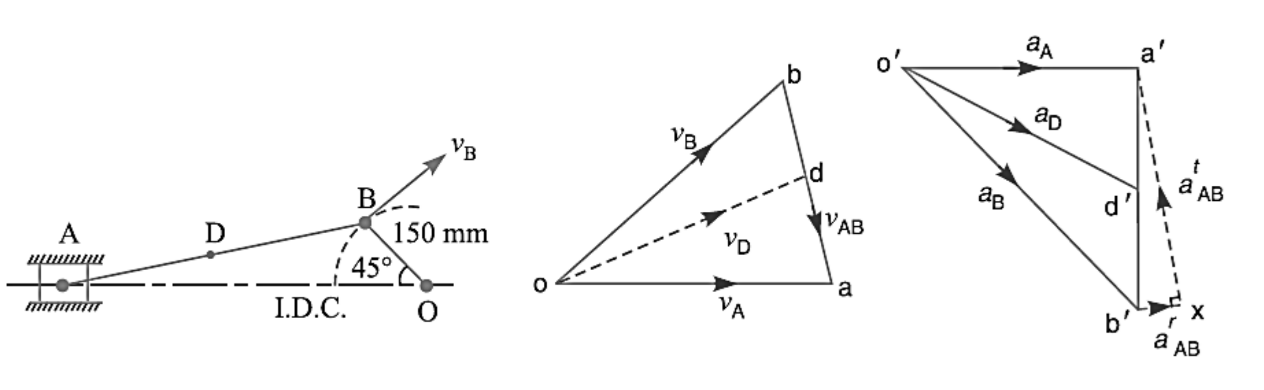
Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B i.e. v_{BO} or vB, such that

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B i.e. v_{AB}, and from point o draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A i.e. v_A. The vectors ba and oa intersect at a.
By measurement, we find the velocity A with respect to B,

$$v_{AB} = vector\ ba = 3.4\, m/s$$

$$v_A = \text{vector oa} = 4 \text{ m/s}$$



(a) Space diagram. (b) Velocity diagram. (c) Acceleration diagram.

In order to find the velocity of the midpoint D of the connecting rod A B, divide the vector ba at d in the same ratio as D divides A B, in the space diagram. In other words,

$$bd/ba = \ BD/BA$$

By measurement, we find that

$$\mathrm{v_D} = \text{vector od} = 4.1 \text{ m/s}$$

We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a^2_{BO} = \ a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \ m/s^2$$

And the radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \ m/s^2$$

$$vector\ o'b' = \ a^r_{BO} = a_B = 148.1 \ m/s^2$$

By measurement, we find that

$$a_D = vector\ o'd' = 117 \ m/s^2$$

We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \ \ rad/s^2$$

From the acceleration diagram, we find that

$$\mathrm{a^t_{AB}} = 103 \ \ \mathrm{m/s^2}$$

We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \ \ rad/s^2$$

Ex. 3.5 An engine mechanism is shown in the figure. The crank CB = 100 mm and the connecting rod BA = 300 mm with the centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and angular acceleration of 1200 rad/s^2. Find:

1. The velocity of G and angular velocity of AB, and
2. Acceleration of G and angular acceleration of AB.

Solution: Given Data:

ω_{BC} = 75 rad/s ; α_{BC} = 1200 rad/s^2, CB = 100 mm = 0.1 m; B A = 300 mm = 0.3 m

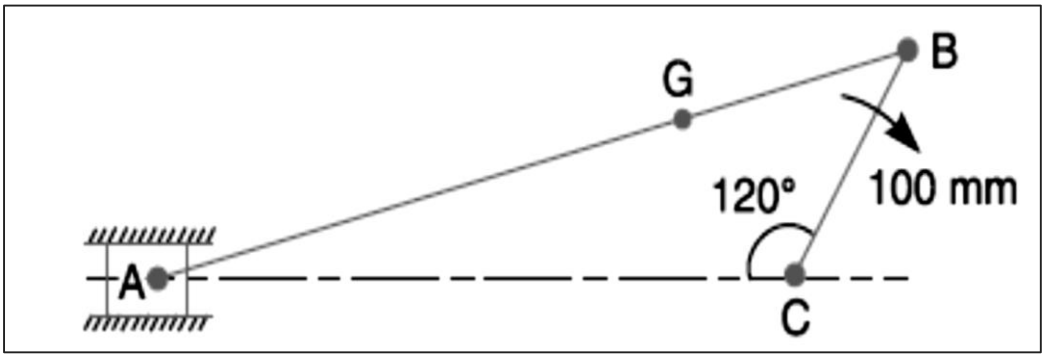
We know that velocity of B with respect to C or velocity of B

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5 \ m/s$$

Since the angular acceleration of the crankshaft, α_{BC} = 1200 rad/s^2, therefore tangential component of the acceleration of B with respect to C,

$$a_{BC}^{t} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120 \ m/s^2$$

$$vector\ cb = v_{BC} = v_B = 7.5\frac{m}{s}$$



By measurement, we find that velocity of G,

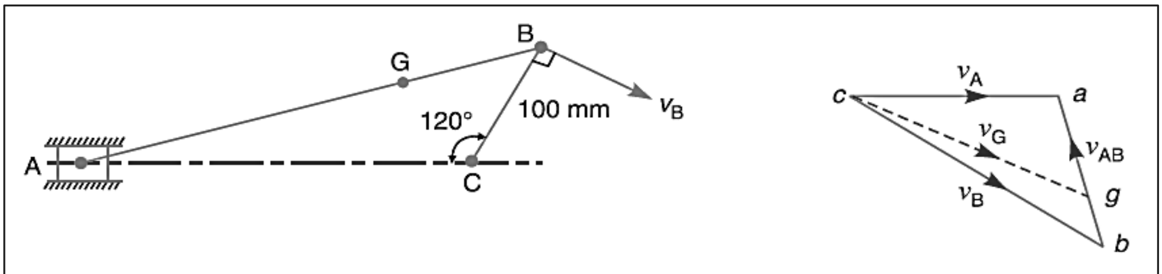
$$v_G = vector\ cg = 6.8\, m/s$$

From the velocity diagram, we find that the velocity of A with respect to B,

$$v_{AB} = vector\ ba = 4\, m/s$$

We know that angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3\, rad/s$$



(a) Space diagram. (b) Velocity diagram.

We know that radial component of the acceleration of B with respect to C

$$a_{BC}^{r} = \frac{v_{BC}^2}{CB} = \frac{(7.5)^2}{0.1} = 562.5 \ m/s^2$$

And a radial component of the acceleration of A with respect to B,

$$a_{AB}^{r} = \frac{v_A^2}{CB} = \frac{(4)^2}{0.3} = 53.3 \ m/s^2$$

$$vector\ c'b'' = a_{BC}^{r} = 562.5\, m/s^2$$

$$vector\ b''b' = a_{BC}^{t} = 120\, m/s^2$$

$$vector\ b'x = a_{AB}^{r} = 53.3\, m/s^2$$

By measurement, we find that acceleration of G,

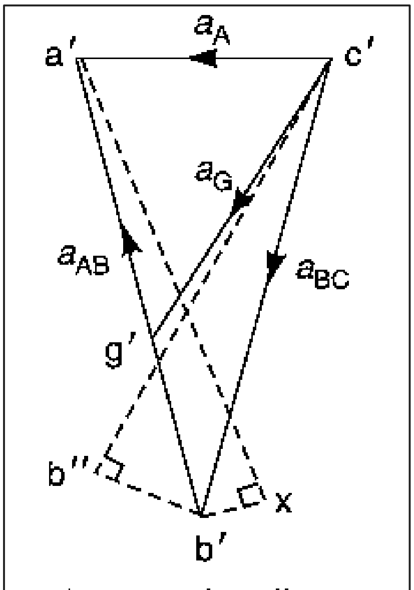
$$a_G = vector\ xa' = 414\, m/s^2$$

From the acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a_{AB}^{t} = vector\ xa' = 546\ m/s^2$$

Angular acceleration of AB,

$$\alpha_{AB} = \frac{a_{AB}^{t}}{BA} = \frac{546}{0.3} = 1820\ rad/s^2$$



(c) Acceleration diagram.

References:

1. Theory of Machines, Rattan S S, Tata McGraw-Hill
2. Theory of Machines, Khurmi R. S., Gupta J. K., S. Chand Publication