4

The Energy Equation & It's Application

Contents

4.1 Momentum & Fluid Flow

Fluid dynamics includes the study of forces causing fluid flow. It provides the methods for studying complex phenomena like weather patterns, high-speed train, flow around objects, blood circulation, etc. Thus dynamics of fluid flow is the study of fluid motion with forces causing the flow. The dynamic behaviour of the flow is analyzed by **Newton's second law of motion**.

Momentum:

In mechanics, the **momentum** of a particle or object is defined as the product of its mass (m) and its velocity (v):

$$Momentum = m \times v$$

e.g. A flying baseball can simply be caught with a glove whereas a moving vehicle is difficult to stop due to its higher momentum.

Newton's Second Law of Motion:

"**Newton's second law** states that the rate of change of the momentum of a body is equal to the net force acting on the body."

$$Net\ Dynamic\ Force = Rate\ of\ change\ of\ momentum\ in\ the\ same\ direction$$

$$\therefore F = \frac{d}{dt}(mv)$$

$$\therefore Fdt = d(mv) \qquad \textit{Eq. (4.1)}$$

Eq. (4.1) is known as the impulse-momentum equation.

If the mass of the body is constant than equation is reduced to

$$F = m\frac{dv}{dt} = ma$$

The momentum equation is used to find the resultant force exerted by a flowing fluid on any surface when the fluid changes its velocity in magnitude or direction or both.

Forces Present in the Fluid Flow:

a) **Body Forces:** Act throughout the entire body of the control volume (such as gravity, centrifugal and electromagnetic forces)
b) **Surface Forces:** act on the control surface (such as pressure and viscous forces and reaction forces at points of contact)

The forces acting on a control volume consist of **body forces** that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and **surface forces** that act on the control surface (such as pressure and viscous forces and reaction forces at points of contact). Only external forces are considered in the analysis.

4.2 Momentum Equation for 2-D Flow along a Streamline

Fig.4.1 shows the two-dimensional problem in which v_1 makes an angle θ with the x-axis, while v_2 makes a corresponding angle $\emptyset$. Since both momentum and force are vector quantities, they can be resolved into components in the x and y directions as shown in *Fig.4.1*.

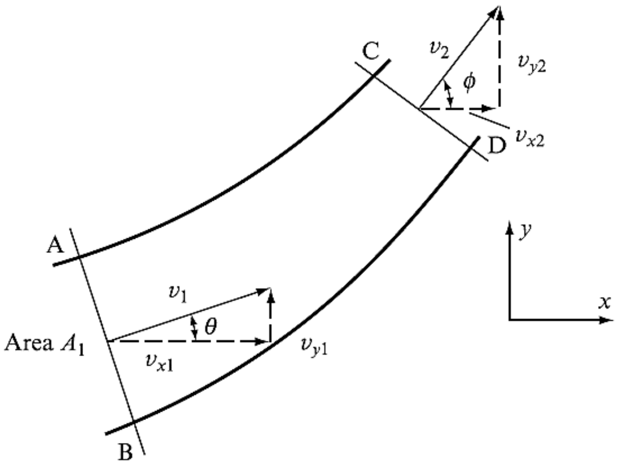

Fig.4.1 – Momentum equation for 2-D flow

Thus if F_x and F_y are the components of the resultant force on the element of fluid ABCD, according to Newton's second law;

$$Force\ exerted\ in\ x\ direction = Rate\ of\ change\ of\ momentum\ in\ x\ direction$$

$$\therefore F_x = Mass\ per\ unit\ time \times Change\ of\ velocity\ in\ x\ direction$$

$$\therefore F_x = \dot{m}(v_2 \cos \emptyset - v_1 \cos \theta) = \dot{m}(v_{x2} - v_{x1})$$

Similarly for y-direction,

$$F_y = \dot{m}(v_2 \sin \emptyset - v_1 \sin \theta) = \dot{m}(v_{y2} - v_{y1})$$

These components can be combined to give resultant force,

$$F = \sqrt{F_x^2 + F_y^2}$$

In general,

$$Total\ force\ exerted\ on\ the\ fluid\ in\ a\ control\ volume\ in\ given\ direction$$
$$= Rate\ of\ change\ of\ momentum\ in\ the\ given\ direction\ of\ the\ fluid\ passing\ through\ the\ control\ volume$$

$$\therefore F_{net} = \dot{m}(v_{out} - v_{in})$$

$$\therefore F_{net} = \dot{m}(v_{out} - v_{in}) \quad \text{Eq. (4.2)}$$

4.2.1 Momentum Correction Factor (β)

It is defined as the ratio of momentum of a flow per second based on actual velocity to the momentum of a flow per second based on average velocity across a section.

Mathematically,

$$\beta = \frac{Momentum\ per\ second\ based\ on\ actual\ velocity}{Momentum\ per\ second\ based\ on\ average\ velocity} \quad \text{Eq. (4.3)}$$

4.2.2 Kinetic Energy Correction Factor (α)

The velocity of flow is assumed to be uniform. In actual flow, the velocity is not uniform across the cross-section and it is larger than what is calculated by the average velocity. In such a case, the kinetic correction factor is applied to the velocity head.

It is defined as the ratio of the kinetic energy of flow per second based on actual velocity across a section to the kinetic energy of flow per second based on average velocity across the same section.

Mathematically,

$$\alpha = \frac{Kinetic\ energy\ per\ second\ based\ on\ actual\ velocity}{Kinetic\ energy\ per\ second\ based\ on\ average\ velocity} \quad \text{Eq. (4.4)}$$

4.3 Euler's Equation of Motion along a Streamline

Consider a streamline in which the flow is taking place in S-direction as shown in *Fig.4.2*.

Take a cylindrical element of cross-sectional area dA and length dS on the streamline.

In Euler's equation of motion, forces due to **gravity** and **pressure** are taken into consideration.

Forces acting on the cylindrical fluid element are:

- Pressure force in the direction of flow,

$$= pdA$$

- Pressure force in the opposite direction of flow,

$$= (p + dp)dA$$

- Weight of element,

$$W = mg = \rho V g = \rho dAdsg$$

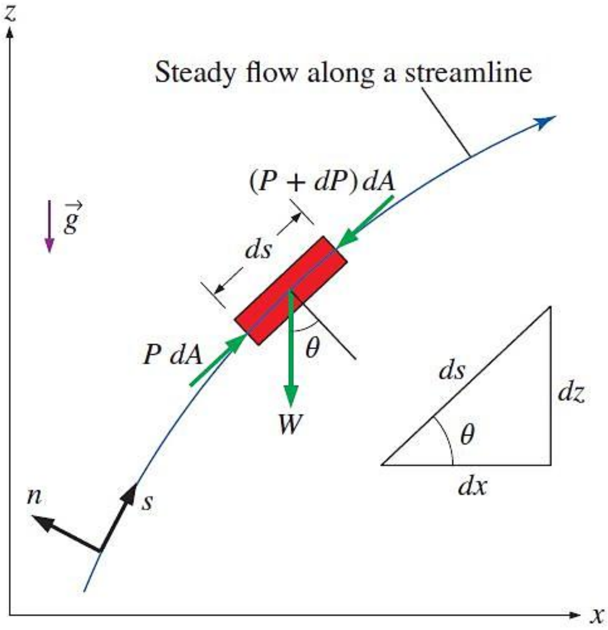

Fig.4.2 – forces acting on the fluid particle

According to Newton's 2nd law of motion, ***"the resultant force on the fluid element in the S-direction must be equal to the product of the mass of fluid and its acceleration in S-direction"***

Therefore,

$$The\ resultant\ forces\ in\ the\ S\ direction = Mass\ of\ fluid\ element \times Acceleration$$

$$\therefore pdA - (p + dP)dA - W \sin\theta = m \times a_s \quad \text{Eq. (4.5)}$$

- As we know that, the velocity of an elementary fluid particle along a streamline is a function of position and time,

$$\therefore v = f(s, t)$$

$$\therefore dv = \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial t} dt$$

Now acceleration,

$$a_s = \frac{dv}{dt} = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}\frac{dt}{dt}$$

For Steady flow,

$$\frac{\partial v}{\partial t} = 0$$

$$\therefore a_s = \frac{dv}{dt} = \frac{\partial v}{\partial s}\frac{ds}{dt} = v\frac{\partial v}{\partial s} \qquad (\because \frac{ds}{dt} = v)$$

Prof. Bhavin J. Vegada, Department of Mechanical Engineering

In a steady flow, the velocity changes w.r.t. position only so, the partial differential becomes the total differential.

$$\therefore a_s = v\frac{dv}{ds} \qquad \textit{Eq. (4.6)}$$

Now from *Eq. (4.5)* and *Eq. (4.6)*

$$\therefore -dPdA - \rho dAdsg \times \frac{dz}{ds} = \rho dAds \times v\frac{dv}{ds}$$

$$\therefore -dP - \rho gdz = \rho vdv$$

$$\therefore \boldsymbol{\frac{dP}{\rho} + vdv + gdz = 0} \qquad \textit{Eq. (4.7)}$$

Eq. (4.7) is known as **Euler's equation of motion**.

4.4 Bernoulli's Equation From Euler's Equation

By integrating Euler's equation of motion,

$$\int \frac{dP}{\rho} + \int vdv + \int gdz = Constant$$

$$\therefore \frac{P}{\rho} + \frac{v^2}{2} + gz = Constant$$

$$\therefore \boldsymbol{\frac{P}{\rho g} + \frac{v^2}{2g} + z = Constant} \qquad \textit{Eq. (4.8)}$$

This is known as **Bernoulli's Equation**. Each term indicates energy per unit weight called Head.

Where,

$\frac{P}{\rho g}$ = Pressure energy per unit weight of fluid or **Pressure Head**

$\frac{v^2}{2g}$ = Kinetic energy per unit weight or **Kinetic Head**

z = Potential energy per unit weight or **Potential Head**

4.4.1 The Statement of Bernoulli's Equation

"It states that in a steady, incompressible and ideal flow, the total energy at any point of the fluid is constant"

The total energy of the fluid consists of:

1) Pressure Energy
2) Kinetic Energy and
3) Potential Energy

4.4.2 Assumptions in Bernoulli's Equation

The following assumptions made in the derivation of Bernoulli's equation:

1) The fluid is ideal i.e. viscosity is zero
2) The flow is steady i.e. flow parameter does not change w.r.t. time
3) The fluid is incompressible i.e. density is constant

4) The flow is irrotational i.e. fluid particle does not rotate about its own axis

5) The flow is 1-D i.e. along a streamline

4.4.3 Bernoulli's Equation for Real Fluids

The Bernoulli's equation is derived based on the assumption that the fluid is ideal, i.e. viscosity is zero. But all real fluids have some viscosity which resists the flow. Thus there are always some losses in fluid flows and hence these losses have to be considered while applying the Bernoulli's equation to the real fluids, and can be written as,

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2+\boldsymbol{h_f}$$

Where,

$h_f = head\ loss\ due\ to\ friction\ between\ two\ section$

4.5 Venturi Meter

A venturi meter is a device used to measure the flow rate of a fluid flowing through the large-sized pipes and for large flow rate.

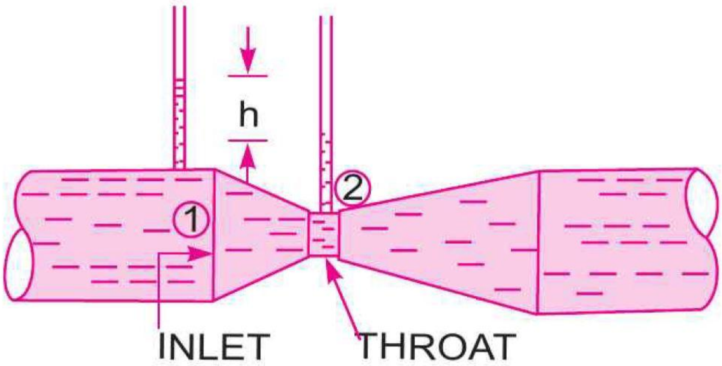

Fig.4.3 – Venturi meter

4.5.1 Construction & Working

It consists of three parts :

1. **A short converging section:**
   In the converging section, the area of flow is decreases and hence the velocity of fluid increases and the static pressure decreases. Normally the convergent angle is of $21° \pm 2°$.

2. **A throat:**
   This is the cylindrical section of the minimum area where the velocity is maximum and pressure is minimum. The throat diameter is usually between 0.25 to 0.5 times the inlet diameter of the pipe. The length of the throat equals its diameter.

3. **A diverging section:**
   The diverging section is a diffuser where the area is increased back to the pipe entrance area and hence the pressure is increased. To recover all the pressure energy, the divergent angle is kept of $5°\ to\ 7°$. This angle has to be kept less so that the flowing fluid has the least tendency to separate from the wall of the pipe. However, with small angles, the length and hence the cost of the venturi meter would increase. So where pressure recovery is not important, the divergent angle may be kept as high as $14°$.

The small-sized venturi meter, suitable for pipelines less than 5cm diameter, are usually made of brass or bronze. The inside surface is smoothly finished to reduce friction.

Large-sized venturi meters are usually made of cast iron with throat made of brass or bronze.

Very large-sized venturi meter, up to 6m pipe diameter has been made of smooth surface concrete. Only the throat is made of machined bronze.

4.5.2 Equation of Rate of Flow-Through Venturi Meter

Consider a venturi meter fitted in a horizontal pipe through which a fluid is flowing, as shown in *Fig.4.3*.

Let,

d_1 = diameter at the inlet of pipe or at section 1

p_1 = pressure at the inlet of pipe or at section 1

v_1 = velocity at the inlet of pipe or at section 1

d_2, p_2, v_2 are corresponding values at the throat or at section 2.

- Applying Bernoulli's equation between section 1 and section 2, we get,

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\therefore \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But the differential head in the piezometric tubes is given by,

$$h = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2)$$

Hence,

$$\therefore h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad \textit{Eq. (4.9)}$$

- Now, applying the Continuity equation between section 1 and section 2, we get,

$$Q = a_1 v_1 = a_2 v_2$$

$$\therefore v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in *Eq. (4.9)*,

$$\therefore h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$\therefore h = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$\therefore v_2^2 = 2gh \times \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore v_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \qquad \textit{Eq. (4.10)}$$

Now,

Discharge,

$$Q = a_2 v_2$$

$$\therefore \boldsymbol{Q} = \frac{\boldsymbol{a_1 a_2}}{\sqrt{\boldsymbol{a_1^2 - a_2^2}}} \times \sqrt{\boldsymbol{2gh}} \qquad \text{Eq. (4.11)}$$

Eq. (4.11) gives discharge under ideal conditions and is called theoretical discharge. The actual discharge will be always less than the theoretical discharge and is given by,

$$\therefore \boldsymbol{Q_{act}} = \boldsymbol{C_d} \frac{\boldsymbol{a_1 a_2}}{\sqrt{\boldsymbol{a_1^2 - a_2^2}}} \times \sqrt{\boldsymbol{2gh}} \qquad \text{Eq. (4.12)}$$

The coefficient of discharge (C_d) for venturi meter is usually in the range of 0.95 to 0.98.

4.5.3 Value of $'h'$ Given by U-tube Manometer

Case-I Heavier fluid in the manometer

If the U-tube manometer contains a liquid which is heavier than the liquid flowing through the pipe. Then,

$$\therefore h = x\left[\frac{S_h}{S_o} - 1\right] = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2) \qquad \text{Eq. (4.13)}$$

Where,

S_h = Specific gravity of heavier fluid used in the manometer

S_o = Specific gravity of fluid flowing through the pipe

x = Difference in the level of heavier fluid in the manometer

Case-II Lighter fluid in the manometer

If the U-tube manometer contains a liquid which is lighter than the liquid flowing through the pipe. Then,

$$\therefore h = x\left[1 - \frac{S_l}{S_o}\right] = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2) \qquad \text{Eq. (4.14)}$$

Where,

S_l = Specific gravity of lighter fluid used in the manometer

S_o = Specific gravity of fluid flowing through the pipe

x = Difference in the level of lighter fluid in the manometer

4.6 Hydraulic Coefficients

4.6.1 Coefficient of Velocity (C_v)

It is defined as the ratio between the actual velocity of a jet at vena-contracta and the theoretical velocity of jet at orifice plate.

$$\therefore \boldsymbol{C_v} = \frac{\boldsymbol{v_{act}}}{\boldsymbol{v_{th}}} \qquad \text{Eq. (4.15)}$$

The theoretical velocity is given by,

$$v_{th} = \sqrt{2gh}$$

The value of C_v varies from 0.95 to 0.99 for different orifices. For sharp-edged orifice generally, it is taken as 0.98.

4.6.2 Coefficient of Contraction (C_c)

It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice.

$$\therefore \boldsymbol{C_c = \frac{a_c}{a_o}} \qquad \textit{Eq. (4.16)}$$

The value of C_c varies from 0.61 to 0.69 for different orifices. In general, it is taken as 0.64.

4.6.3 Coefficient of Discharge (C_d)

It is defined as the ratio of the actual discharge to the theoretical discharge from an orifice.

$$\therefore \boldsymbol{C_d = \frac{Q_{act}}{Q_{th}}} \qquad \textit{Eq. (4.17)}$$

$$\therefore C_d = \frac{a_c \times v_{act}}{a_o \times v_{th}}$$

$$\therefore \boldsymbol{C_d = C_c \times C_v} \qquad \textit{Eq. (4.18)}$$

The value of C_d varies from 0.61 to 0.65 for different orifices. In general, it is taken as 0.62.

4.7 Orifice Meter

An orifice meter is a device used to measure the flow rate of a fluid flowing through the pipe.

It is a cheaper device compared to the venturi meter.

4.7.1 Construction & Working

It consists of a flat circular plate that has a circular sharp-edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

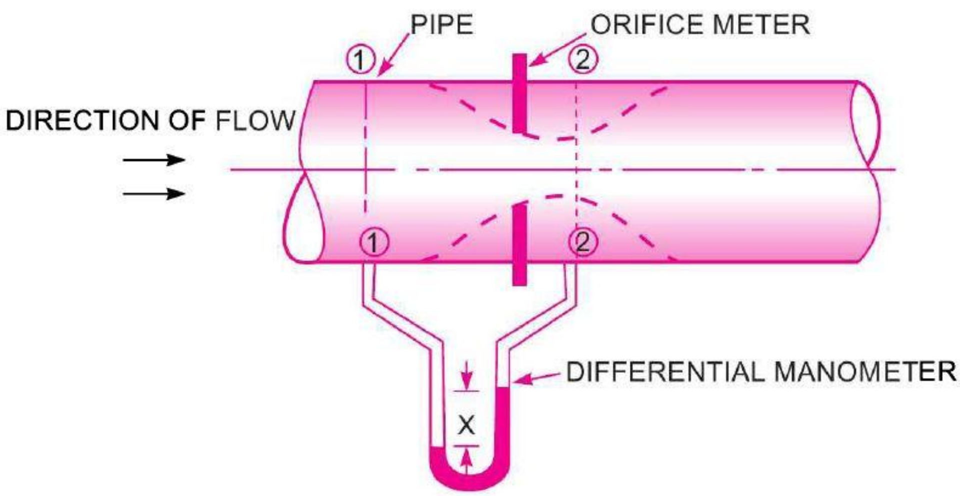

Fig.4.4 – Orifice meter

In orifice, minimum cross-section occurs downstream from the orifice plate. The section of the minimum area is called **vena-contracta**. Minimum pressure and maximum velocity occur in this section.

A differential manometer is connected at section 1, which is at a distance of about 1.5 to 2 times the pipe diameter upstream from the orifice plate and at section 2, i.e. vena-contracta, which is at a distance 0.5 times the diameter of the orifice on the downstream side from the orifice plate as shown in *Fig.4.4*.

4.7.2 Equation of Rate of Flow-Through Orifice Meter

Consider an orifice plate fitted in a horizontal pipe through which a fluid is flowing, as shown in *Fig.4.4*.

Let,

d_o = diameter of the orifice

d_1 = diameter at the inlet of pipe or at section 1

p_1 = pressure at the inlet of pipe or at section 1

v_1 = velocity at the inlet of pipe or at section 1

d_2, p_2, v_2 are corresponding values at the vena-contracta or at section 2

$C_c = \frac{a_c}{a_o}$ = Coefficient of contraction

$C_v = \frac{v_{act}}{v_{th}}$ = Coefficient of velocity

$C_d = \frac{Q_{act}}{Q_{th}} = C_c \times C_v$ = Coefficient of discharge

- Applying Bernoulli's equation between section 1 and section 2, we get,

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\therefore \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But the differential head in the piezometric tubes is given by,

$$h = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2)$$

Hence,

$$\therefore h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad \textit{Eq. (4.19)}$$

- Now, applying the Continuity equation between section 1 and section 2, we get,

$$Q = a_1 v_1 = a_2 v_2$$

$$\therefore v_1 = \frac{a_2 v_2}{a_1}$$

$$\therefore v_1 = \frac{C_c a_o v_2}{a_1} \qquad \{from\ Eq. (4.16)\}$$

Substituting this value of v_1 in *Eq. (4.19)*,

$$\therefore h = \frac{v_2^2}{2g} - \frac{\left(\frac{C_c a_o v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]$$

$$\therefore v_2^2 = \frac{2gh}{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}$$

$$\therefore v_2 = \frac{\sqrt{2gh}}{\sqrt{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}} \qquad \textit{Eq. (4.20)}$$

Now,

- Discharge,

$$Q = a_2 v_2 = C_c a_o v_2$$

Put the value of v_2 from *Eq. (4.20)*, we get,

$$\therefore Q = C_c a_o \times \frac{\sqrt{2gh}}{\sqrt{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}} \qquad \textit{Eq. (4.21)}$$

- **To find the value of C_c:**

From *Eq. (4.15)*,

$$C_v = \frac{v_{act}}{v_{th}} = \frac{v_{2act}}{v_{2th}}$$

Actual velocity is given by *Eq. (4.20)*,

$$\therefore v_{2act} = \frac{\sqrt{2gh}}{\sqrt{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}}$$

And for theoretical velocity $C_c = 1$,

$$\therefore v_{2th} = \frac{\sqrt{2gh}}{\sqrt{\left[1 - \left(\frac{a_o}{a_1}\right)^2\right]}}$$

Hence,

$$C_v = \frac{\dfrac{\sqrt{2gh}}{\sqrt{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}}}{\dfrac{\sqrt{2gh}}{\sqrt{\left[1 - \left(\frac{a_o}{a_1}\right)^2\right]}}} = \frac{\sqrt{\left[1 - \left(\frac{a_o}{a_1}\right)^2\right]}}{\sqrt{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}}$$

Therefore,

$$C_c = \frac{C_d}{C_v} = C_d \times \frac{\sqrt{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}}{\sqrt{\left[1 - \left(\frac{a_o}{a_1}\right)^2\right]}}$$

- Put the value of C_c in *Eq. (4.21)*, we get,

$$\therefore Q = C_d \times \frac{\sqrt{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}}{\sqrt{\left[1 - \left(\frac{a_o}{a_1}\right)^2\right]}} \times a_o \times \frac{\sqrt{2gh}}{\sqrt{\left[1 - C_c^2\left(\frac{a_o}{a_1}\right)^2\right]}}$$

$$\therefore \boldsymbol{Q_{act} = C_d \frac{a_1 a_o}{\sqrt{a_1^2 - a_o^2}} \times \sqrt{2gh}} \qquad \text{Eq. (4.22)}$$

Eq. (4.22) gives the actual discharge through the orifice meter.

4.8 Rotameter – Variable Area Flowmeter

A simple, reliable, inexpensive, and easy-to-install flowmeter with reasonably low-pressure drop and no electrical connections that gives a direct reading of flow rate for a wide range of liquids and gases is the **variable area flowmeter**, also called a **rotameter** or **float meter**.

A variable-area flowmeter consists of a vertical tapered conical transparent tube made of glass or plastic with a float inside that is free to move, as shown in *Fig.4.5*.

As fluid flows through the tapered tube, the float rises within the tube to a location where the float weight, drag force, and buoyancy force balance each other and the net force acting on the float is zero.

The flow rate is determined by simply matching the position of the float against the graduated flow scale outside the tapered transparent tube. The float itself is typically either a sphere or a loose-fitting piston-like cylinder.

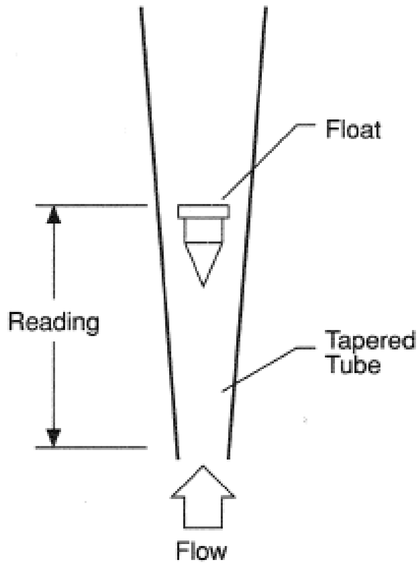

Fig.4.5 - Rotameter

The weight and the buoyancy force acting on the float are constant, but the drag force changes with the flow velocity. Also, the velocity along the tapered tube decreases in the flow direction because of the increase in the cross-sectional area.

There is a certain velocity that generates enough drag to balance the float weight and the buoyancy force, and the location at which this velocity occurs around the float is the location where the float settles.

4.9 Notches & Weirs

Notch: A notch is a device used for measuring the discharge of a liquid through a small channel or a tank.

Weir: A weir is a concrete or masonry structure, placed in an open channel over which the flow occurs.

Difference Between Notch & Weir:

- The notch is of small size while the weir is of a bigger size.
- The notch is used to measuring the discharge in small channels, while the weir is used to measure the discharge in large bodies like river or dam.
- The notch is generally made of the metallic plate while weir is made of concrete or masonry structure.

Nappe or Vein: The sheet of water flowing through a notch or over a weir is called nappe or vein.

Crest or Sill: The bottom edge of a notch or the top of a weir over which the water flows, is known as sill or crest.

4.9.1 Classification of Notches

The notches are classified as:

1) According to the shape of the opening
    a. Rectangular Notch
    b. Triangular Notch
    c. Trapezoidal Notch
    d. Stepped Notch
2) According to the effect of the sides on the nappe
    a. Notch with end contraction
    b. Notch without end contraction or suppressed notch

4.9.2 Classification of Weirs

The weirs are classified as:

1) According to the shape of the opening
    a. Rectangular weir
    b. Triangular weir and
    c. Trapezoidal weir or Cippoletti weir
2) According to the shape of the crest
    a. Sharp-crested weir
    b. Broad-crested weir
    c. Narrow-crested weir and
    d. Ogee-shaped weir
3) According to the effect of sides on the emerging nappe
    a. Weir with end contraction and
    b. Weir without end contraction

4.9.3 Discharge over a Rectangular Notch

Consider a rectangular notch or weir provided in a channel carrying water as shown in *Fig.4.6*.

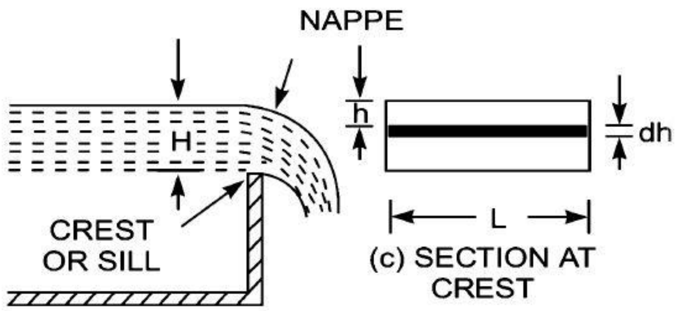

Fig.4.6 – Rectangular Notch

Let,

$H =$ Head of water over the crest

$L =$ Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of thickness dh and length L at a depth h from the free surface of the water as shown in *Fig.4.6*.

The discharge through the strip,

$$dQ = Area\ of\ Strip \times Theoretical\ Velocity$$

$$\therefore dQ = (L \times dh) \times \sqrt{2gh}$$

The total discharge,

$$Q = \int_0^H dQ = \int_0^H (L \times dh) \times \sqrt{2gh}$$

$$\therefore Q = L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$\therefore Q = L \times \sqrt{2g} \left[\frac{h^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^H$$

$$\therefore Q = L \times \sqrt{2g} \left[\frac{H^{3/2}}{3/2} \right]$$

$$\therefore Q = \frac{2}{3} L \times \sqrt{2g} \times H^{3/2}$$

The actual discharge is given by,

$$\therefore \boldsymbol{Q_{act} = \frac{2}{3}\, C_d \times L \times \sqrt{2g} \times H^{3/2}} \qquad \textit{Eq. (4.23)}$$

4.9.4 Discharge Over a Triangular Notch

The triangular notch is also known as a V-notch. Consider a triangular notch or weir provided in a channel carrying water as shown in *Fig.4.7*.

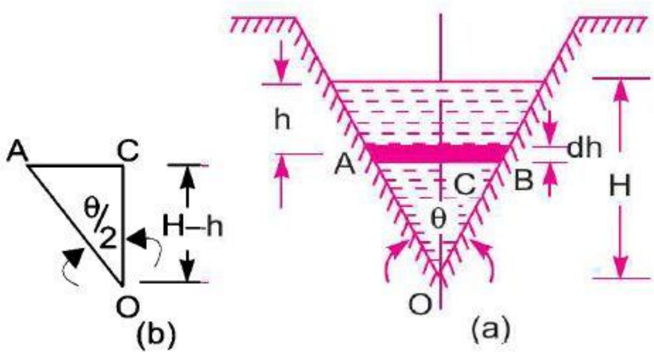

Fig.4.7 – Triangular Notch

Let,

H = Head of water over a triangular notch

θ = Angle of a notch

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of thickness dh at a depth h from the free surface of the water as shown in *Fig.4.7*.

The discharge through the strip,

$$dQ = Area\ of\ Strip \times Theoretical\ Velocity$$

$$\therefore dQ = (AB \times dh) \times \sqrt{2gh} \qquad \textit{Eq. (4.24)}$$

From *Fig.4.7* (b),

$$\tan\frac{\theta}{2} = \frac{AC}{OC}$$

$$\therefore AC = \tan\frac{\theta}{2} \times OC$$

$$\therefore AC = \tan\frac{\theta}{2} \times (H - h)$$

Width of the strip,

$$AB = 2AC = 2 \times \tan\frac{\theta}{2} \times (H - h)$$

From *Eq. (4.24)*,

$$dQ = (AB \times dh) \times \sqrt{2gh}$$

$$\therefore dQ = 2 \times \tan\frac{\theta}{2} \times (H - h) \times \sqrt{2gh}\, dh$$

$$\therefore dQ = 2 \times \tan\frac{\theta}{2} \times \sqrt{2g} \times \left(H\, h^{1/2} - h\, h^{1/2}\right) dh$$

The total discharge,

$$Q = \int_0^H dQ = \int_0^H 2 \times \tan\frac{\theta}{2} \times \sqrt{2g} \times \left(H\, h^{1/2} - h\, h^{1/2}\right) dh$$

$$\therefore Q = 2 \times \tan\frac{\theta}{2} \times \sqrt{2g} \int_0^H \left(H\, h^{1/2} - h^{3/2}\right) dh$$

$$\therefore Q = 2 \times \tan\frac{\theta}{2} \times \sqrt{2g} \left[\frac{H h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$\therefore Q = 2 \times \tan\frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2}\right]$$

$$\therefore Q = 2 \times \tan\frac{\theta}{2} \times \sqrt{2g} \left[\frac{4}{15} H^{5/2}\right]$$

$$\therefore \boldsymbol{Q = \frac{8}{15} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}} \qquad \textit{Eq. (4.25)}$$

The actual discharge through triangular notch is given by,

$$\therefore \boldsymbol{Q_{act} = \frac{8}{15}\, C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}} \qquad \textit{Eq. (4.26)}$$

4.9.5 Advantages of Triangular Notch or Weir Over Rectangular Notch or Weir

1. For measuring low discharge, a triangular notch gives more accurate results than a rectangular notch because the height of water in a V-notch is more as compared to a rectangular notch.

2. In case of a triangular notch, only one reading i.e. H is required to find the discharge.
3. Ventilation of a triangular notch is not necessary.

4.9.6 Discharge Over a Trapezoidal Notch

As shown in *Fig.4.8*, a trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular notch and the discharge through a triangular notch.

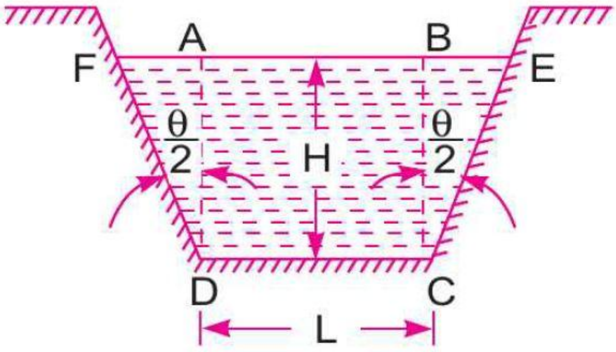

Fig.4.8 – Trapezoidal Notch

Let,

H = Head of water over a notch

L = Length of the crest of the notch

C_{d_1} = Coefficient of discharge for the rectangular portion

C_{d_2} = Coefficient of discharge for the triangular portion

The discharge through a trapezoidal notch or weir is given by,

$$Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \qquad \text{Eq. (4.27)}$$

Cippoletti Weir

The Cippoletti weir is a trapezoidal weir which has side slope of 1:4 (1 horizontal & 4 vertical). Such slope increases the discharge through the triangular portion of the weir. Hence such trapezoidal weir does not have end contractions.

End Contractions: The weir with end contraction means, the width of nappe is less than the width of the weir. In such a case, the correction factors are to be applied to the calculated discharge for finding accurate results.

4.10 References

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