5

Dimensional Analysis & Similarities

Contents

5.1 Introduction to Dimensional Analysis

Dimensional Analysis is a method of dimensions. It is a mathematical technique used in research work for design and for conducting model tests.

Need for Dimensional Analysis & Similarities

The analytical tools available to solve the momentum and energy equations of complex flow problems are not capable to give accurate results, particularly in turbulent and separating flows. Therefore, solutions to such problems are mostly determined from experiments.

In any physical phenomenon, there are many variables are involved. In order to reduce the number of experiments, these variables should be as minimum as possible. The dimensional analysis is a method for reducing the number and complexity of experimental variables and it will form the dimensionless numbers. We know that the most popular dimensionless group in fluid mechanics is the Reynolds number.

Also, experimental techniques are quite expensive and time-consuming. Many of the times it is not possible to perform experiments in the laboratory on the actual prototype. Therefore, experiments are usually carried out on the models. But, can we apply the data gathered from an experiment on a model to the design of a full-scale device? The dimensional analysis and similitude (similarity between model and prototype) help to answer the above question.

Now, no need to build actual aircraft, ship, dam or any large machine before tests are carried out on small scale models in wind tunnel.

Application of Dimensional Analysis

- To derive a formula that represents the relationship between the physical quantities that affect a given physical phenomenon.
- To check the dimensional homogeneity of any equation of fluid flow.
- To provide scaling laws that can convert data from small models to large models.
- To develop a dimensionless number which is useful to compare the different problems.

Dimensions & Units

A **Dimension** is a measure of a physical quantity, while a **Unit** is expressing a dimension.

For example, length is a dimension that is measured in units such as microns, feet, meter, etc.

Fundamental Dimensions

All physical quantities are measured by comparison. This comparison is always made with respect to some arbitrarily fixed value.

In fluid mechanics, **Mass (M), Length (L) and Time (T)** are called fundamental dimensions or primary quantities, which are independent of each other.

Apart from this, in heat transfer, **Temperature (θ)** is used as a fundamental dimension.

Derived Dimensions

The quantities, which are expressed in terms of the fundamental quantities are known as derived or secondary quantities. For example, the force can be expressed in terms of mass, length and time as below:

The dimension of force: $Force = m \times a = mass \times \frac{length}{time^2} = \frac{kg \times m}{s^2}$

Kinematic and Dynamic Quantities

The kinematic quantities describe the fluid motion, whereas dynamic quantities cause that fluid motion.

Dimensional Constant

It is quantities, which remains constant for a given case. It may vary from case to case.

Dimensional Variable

It is quantities which vary during a given case.

Dependent & Independent Variable

The dependent variable is one, whose value is to be determined. All other variables which affect the value of the dependent variable are called **Independent variables**.

For example,

$$h_f = \frac{4fLv^2}{2gD}$$

Here, h_f is to be determined for a given value of f, L, v, g, D and hence, h_f is a dependent variable whereas, f, L, v, g, D are independent variables.

*Table **5.1** - Symbols and dimensions of quantities used in fluid mechanics*

Sr. No.	Physical Quantity	Symbol	Unit (SI System)	Dimension (MLT System)
A	**Fundamental Quantities**			
1	Mass	m	$Kilogram\ (kg)$	$M^1L^0T^0$
2	Length	l	$Meter\ (m)$	$M^0L^1T^0$
3	Time	t	$Second\ (sec)$	$M^0L^0T^1$
4	Temperature	T	$Kelvin\ (K)$	$M^0L^0T^0\theta^1$
B	**Derived Quantities**			
I	**Geometric Quantities**			
1	Area	A	m^2	$M^0L^2T^0$
2	Volume	V	m^3	$M^0L^3T^0$
3	Moment of Inertia	I	m^4	$M^0L^4T^0$
4	Roughness	ε	m	$M^0L^1T^0$
II	**Kinematic Quantities**			
1	Velocity	v	m/sec	$M^0L^1T^{-1}$
2	Angular Velocity	ω	rad/sec	$M^0L^0T^{-1}$

3	Acceleration	a	m/sec^2	$M^0L^1T^{-2}$
4	Angular Acceleration	α	rad/sec^2	$M^0L^0T^{-2}$
5	Acceleration due to Gravity	g	m/sec^2	$M^0L^1T^{-2}$
6	Rotational Speed	N	rev/min	$M^0L^0T^{-1}$
7	Kinematic Viscosity	ν	m^2/sec	$M^0L^2T^{-1}$
8	Discharge	Q	m^3/sec	$M^0L^3T^{-1}$
III	**Dynamic Quantities**			
1	Force/Weight/Resistance/Thrust	$F/W/R/T$	N	$M^1L^1T^{-2}$
2	Specific Weight	w	N/m^3	$M^1L^{-2}T^{-2}$
3	Pressure	p	N/m^2	$M^1L^{-1}T^{-2}$
4	Shear Stress	τ	N/m^2	$M^1L^{-1}T^{-2}$
5	Modulus of Elasticity	E/K	N/m^2	$M^1L^{-1}T^{-2}$
6	Surface Tension	σ	N/m	$M^1L^0T^{-2}$
7	Density	ρ	kg/m^3	$M^1L^{-3}T^0$
8	Dynamic Viscosity	μ	$N \times sec/m^2$	$M^1L^{-1}T^{-1}$
9	Work/Energy	W/E	$Joule\ or\ (N \times m)$	$M^1L^2T^{-2}$
10	Power	P	$Watt\ or\ J/sec$	$M^1L^2T^{-3}$
11	Torque	T	$N \times m$	$M^1L^2T^{-2}$
12	Momentum	M	$kg \times m/sec$	$M^1L^1T^{-1}$

5.2 Dimensional Homogeneity

The law of dimensional homogeneity is stated as,

"Every additive term in an equation must have the same dimensions."

Thus if the dimensions of each term on both sides of an equation are the same, the equation is called the **dimensionally homogeneous equation**.

Thus, the power of fundamental dimensions (M, L, T) on both sides of the equation will be identical for a dimensionally homogeneous equation. Such equations are independent of the system of units.

For Example,

Consider the equation of velocity,

$$v = \sqrt{2gh}$$

The dimension of L.H.S.,

$$v = \frac{length}{time} = M^0 L^1 T^{-1}$$

The dimension of R.H.S.,

$$\sqrt{2gh} = \sqrt{\frac{L}{T^2} \times L} = \frac{L}{T} = M^0 L^1 T^{-1}$$

Here, the dimension of L.H.S. = the dimension of R.H.S. = $M^0 L^1 T^{-1}$

Therefore, $v = \sqrt{2gh}$ is a dimensionally homogeneous equation.

5.2.1 Application of Dimensional Homogeneity

- It helps to check whether the equation is dimensionally homogeneous or not.
- To determine the dimensions of a physical quantity.
- It helps to convert the unit from one system to another system.
- To help in dimensional analysis and model testing.

5.3 Methods of Dimensional Analysis

With the help of dimensional analysis, the equation of physical phenomenon can be developed in terms of dimensionless numbers and the number of variables can be reduced. Based on the principle of dimensional homogeneity, there are two methods of dimensional analysis are important as follows:

1. Rayleigh's Method, also known as Indicial Method
2. Buckingham's π-theorem, also known as Group Method

5.3.1 Rayleigh's Method

Rayleigh's method is used for determining the expression for a dependent variable which depends upon a maximum of three or four variables.

If the number of independent variables increases more than four, it becomes difficult to find the expression for the dependent variable.

Steps:

1. Let X is a dependent variable, which depends on $X_1, X_2, X_3 \dots \dots X_n$ variables. Then according to Rayleigh's method, the expression can be written as,

$$X = f(X_1, X_2, X_3 \dots \dots X_n)$$

2. Now, introduce constant K and also put power on independent variables,

$$\therefore X = K(X_1^a, X_2^b, X_3^c, \dots \dots X_n^n)$$

3. Express dependent and independent variables in terms of fundamental dimensions M, L and T.
4. The value of power $a, b, c \dots \dots n$ can be found by comparing the power of fundamental dimensions of both sides.
5. Hence the expression for the dependent variable can be obtained.

5.3.2 Buckingham's π-theorem

Rayleigh's method of dimensional analysis becomes more laborious if the independent variables are more than the no. of fundamental dimensions (M, L, T).

This difficulty is overcome by using Buckingham's π-theorem.

Statement of Buckingham's π-theorem

It states that,

"If there are n no. of variables (dependent & independent) and m no. of fundamental dimensions (M, L, T) in any physical phenomenon then the variables are arranged into $(n-m)$ no. of dimensionless terms. Each term is called π-term."

Steps:

1. Let $X_1, X_2, X_3 \dots \dots X_n$ are the variables involved in a physical problem. Where X_1 depends on $X_2, X_3 \dots \dots X_n$.

$$\therefore X_1 = f(X_2, X_3, \dots \dots X_n) \qquad \textit{Eq. (5.1)}$$

   It can also be written as,

$$\therefore f_1(X_1, X_2, X_3, \dots \dots X_n) = 0 \qquad \textit{Eq. (5.2)}$$

2. If there are n variables with m no. of fundamental dimensions, then *Eq. (5.2)* can be written in terms of π-terms as

$$\therefore f_1(\pi_1, \pi_2, \dots \dots \pi_{n-m}) = 0 \qquad \textit{Eq. (5.3)}$$

3. Each π-term contains $m+1$ variables in which m variables are repeating variables. Let X_2, X_3 and X_4 are repeating variables if the fundamental dimensions are 3.
4. Hence π-terms can be written as,

$$\pi_1 = X_2^{a_1}, X_3^{b_1}, X_4^{c_1} X_1$$

$$\pi_2 = X_2^{a_2}, X_3^{b_2}, X_4^{c_2} X_5$$

   And so on but up to

$$\pi_{n-m} = X_2^{a_{n-m}}, X_3^{b_{n-m}}, X_4^{c_{n-m}} X_n$$

5. The above equations are solved by the principle of dimensional homogeneity and the values of powers are obtained.
6. These values are substituted in π-terms and the π-terms are substituted in *Eq. (5.3)*.
7. The final equation for the given problem is obtained by expressing any one of the π-term as a function of others as

$$\therefore \pi_1 = \phi(\pi_2, \pi_3 \dots \dots \pi_{n-m}) \qquad \textit{Eq. (5.4)}$$

5.3.2.1 Method of Selecting Repeating Variables

The variables which are repeated in each π-term are called **Repeating variables**. The no. of repeating variables is equal to the no. of fundamental dimensions of the problem. The choice of repeating variables is made on the following considerations:

1. The dependent variable should not be selected as a repeating variable.
2. The repeating variable should be selected in such a way that one variable contains geometric property (Length, Diameter, etc), another variable contains flow or kinematic property (Velocity, Acceleration, etc.) and the third variable contains fluid or dynamic property (Density, Viscosity, etc.).
3. The repeating variables selected should not form a dimensionless group.
4. The repeating variables together must have the same no. of fundamental dimensions. i.e. if total no. of fundamental dimensions are 3 then repeating variables together must have all 3 fundamental dimensions.
5. No two repeating variables should have the same dimensions.

Note:
In most of the fluid mechanics problems, the choice of repeating variables may be
(i) ρ, v, d or ρ, v, l
(ii) μ, v, d or μ, v, l

5.4 Dimensionless Numbers & It's Significance

The fluid is generally subjected to various forces such as

a) Inertia Force (Due to its self mass)
b) Viscous Force (Due to viscosity)
c) Gravity Force (Due to gravitational attraction)
d) Pressure Force (Due to difference in pressure)
e) Capillary Force (Due to surface tension)
f) Compressibility/Elasticity Force (Due to elasticity)

For a flowing fluid, the above-listed forces may not always be present. Also, the forces which are present in a fluid flow problem are not of equal magnitude. There are always one or two forces that dominate the other forces and these dominating forces govern the flow of fluid.

The ratio of any two forces is a dimensionless number. The most important dimensionless numbers are:

a) Reynolds Number (Re)
b) Froude's Number (Fr)
c) Euler's Number (Eu)
d) Weber's Number (We)
e) Mach Number (M)

5.4.1 Reynolds Number (Re)

Reynolds number is the ratio of inertia force to the viscous force.

Inertia Force,	Viscous Force,	Reynolds Number,
$F_i = m \times a$ $= \rho V \times \frac{v}{t}$ $= \rho AL \times \frac{v}{t}$ $= \rho L^2 \times \frac{L}{t} \times v$ $\therefore F_i = \rho v^2 L^2$	$F_v = \tau \times A$ $= \mu \frac{du}{dy} \times A$ $= \mu \frac{v}{L} \times L^2$ $\therefore F_v = \mu v L$	$Re = \frac{F_i}{F_v}$ $= \frac{\rho v^2 L^2}{\mu v L}$ $\therefore Re = \frac{\rho v L}{\mu} = \frac{vL}{\nu}$

Significance:

- Used in fluid flow problems where viscous forces alone are predominant.
- If the **Viscous Force** is of prime importance, the dynamic similarity is said to exist between the model and prototype hence **Reynolds Number** of model and prototype should be the same.
- Re measures the relative magnitude of the inertia force to viscous force occurring in the flow.
- Smaller the Re, the greater the viscous effect and flow will become laminar.
- Higher the Re, greater the inertia effect and flow will become turbulent.
- In a **Pipe flow,**

  If Re < 2000 ☞ Laminar Flow

  If 2000 < Re < 4000 ☞ Transition between Laminar and Turbulent

  If Re > 4000 ☞ Turbulent Flow

Examples:

- The flow of incompressible fluid in a closed pipe
- Resistance experienced by submarines, aeroplanes, fully immersed bodies, etc.

5.4.2 Froude's Number (Fr)

Froude's number is the square root of the ratio of inertia force to the gravitational force.

Inertia Force,	Gravitational Force,	Froude's Number,
$F_i = m \times a$ $= \rho V \times \frac{v}{t}$ $= \rho AL \times \frac{v}{t}$ $= \rho L^2 \times \frac{L}{t} \times v$ $\therefore F_i = \rho v^2 L^2$	$F_g = m \times g$ $= \rho V \times g$ $\therefore F_g = \rho L^3 g$	$Fr = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho v^2 L^2}{\rho L^3 g}}$ $\therefore Fr = \frac{v}{\sqrt{Lg}}$

Significance:

- Used in fluid flow problems where gravitational forces alone are predominant.
- If the **Gravitational Force** is of prime importance, the dynamic similarity is said to exist between the model and prototype hence **Froude's Number** of models and prototype should be the same.

Examples:

- Froude's number is very much significant for flows with free surface effects such as in case of flow over spillways, sluices, open-channel flow, etc.
- It is also used where waves are likely to be formed on surface i.e. motion of the ship in a rough and turbulent sea.

5.4.3 Euler's Number (Eu)

Euler's number is the square root of the ratio of inertia force to the Pressure Force.

Inertia Force,	Pressure Force,	Euler's Number,
$F_i = m \times a$ $= \rho V \times \frac{v}{t}$ $= \rho A L \times \frac{v}{t}$ $= \rho L^2 \times \frac{L}{t} \times v$ $\therefore F_i = \rho v^2 L^2$	$F_p = p \times A$ $\therefore F_p = p L^2$	$Eu = \sqrt{\frac{F_i}{F_p}} = \sqrt{\frac{\rho v^2 L^2}{p L^2}}$ $\therefore Eu = \frac{v}{\sqrt{\frac{p}{\rho}}}$

Significance:

- Used in fluid flow problems where pressure forces alone are predominant.
- If the **Pressure Force** is of prime importance, the dynamic similarity is said to exist between the model and prototype hence **Euler's Number** of models and prototype should be the same.

Examples:

- Flow-through a closed pipe
- Discharge through an orifice, sluices, etc.
- Water hammer created in the penstock
- Pressure rise due to sudden closure of valves

5.4.4 Weber's Number (We)

Weber's number is the square root of the ratio of inertia force to the Surface Tension Force.

Inertia Force,	Surface Tension Force,	Weber's Number,
$F_i = m \times a$ $= \rho V \times \frac{v}{t}$ $= \rho A L \times \frac{v}{t}$ $= \rho L^2 \times \frac{L}{t} \times v$ $\therefore F_i = \rho v^2 L^2$	$F_s = \sigma \times L$	$We = \sqrt{\frac{F_i}{F_s}} = \sqrt{\frac{\rho v^2 L^2}{\sigma L}}$ $\therefore We = \frac{v}{\sqrt{\frac{\sigma}{\rho L}}}$

Significance:

- Used in fluid flow problems where surface tension forces alone are predominant.
- If the **Surface Tension Force** is of prime importance, the dynamic similarity is said to exist between the model and prototype hence **Weber's Number** of models and prototype should be the same.

Examples:

- Capillary tube action
- A very thin sheet of liquid flowing over a surface
- The flow of blood in veins and arteries

5.4.5 Mach Number (M)

Mach number is the square root of the ratio of inertia force to the Elasticity Force. It is also defined as the ratio of the velocity of the fluid to the velocity of sound.

Inertia Force,	Elasticity Force,	Weber's Number,
$F_i = m \times a$ $= \rho V \times \frac{v}{t}$ $= \rho AL \times \frac{v}{t}$ $= \rho L^2 \times \frac{L}{t} \times v$ $\therefore F_i = \rho v^2 L^2$	$F_e = K \times A$ $\therefore F_e = K \times L^2$	$M = \sqrt{\frac{F_i}{F_e}} = \sqrt{\frac{\rho v^2 L^2}{K L^2}}$ $\therefore M = \frac{v}{\sqrt{\frac{K}{\rho}}} = \frac{v}{C}$ $Where, C = velocity\ of\ sound$

Significance:

- If the **Elasticity Force** is of prime importance, the dynamic similarity is said to exist between the model and prototype hence **Mach Number** of models and prototype should be the same.
- Predominantly used in problems in which fluid compressibility is important.
- If,

$$v \gg C \Rightarrow M \gg 1 \Rightarrow Hypersonic\ Flow$$

$$v > C \Rightarrow M > 1 \Rightarrow Supersonic\ Flow$$

$$v = C \Rightarrow M = 1 \Rightarrow Sonic\ Flow$$

$$v < C \Rightarrow M < 1 \Rightarrow Subsonic\ Flow$$

- It is used in the compressible flow (where density variation is significant).

Examples:

- Aerodynamic testing (e.g. Missiles)
- Water hammer effects
- The flow of gases exceeding sound velocity

5.5 Model Analysis

Engineers are always engaged in the design of hydraulic structures (Dam, Spillways, etc.) or hydraulic machines (Turbine, Pump, etc.). Actually, they want to find out in advance, how the actual structure or machine would behave when it is actually constructed.

For this purpose, they need to do experiments, but experiments can not be carried out on the full-size structure or machines. Then it is necessary to construct a small scale replica of the structure or machines and tests are performed on it.

Prototype: The actual structure or machine is called a prototype.

Model: It is the small scale replica of the actual structure or machine.

> ***Note:*** *It is not necessary that the models should be smaller than the prototype (though in most of cases it is), they may be larger than the prototype.*

Model Analysis: The study of models of actual structure or machine is called model analysis. It is actually an experimental method of finding solutions for complex flow problems.

Advantages of Model Analysis:

- The performance of the hydraulic structure or machine can be easily predicted, in advance from its model.
- Model tests are economical and convenient because the design, construction, and operation of the model can be easily varied in no. of times if required, till all the defects of the model are eliminated, and efficient and suitable design obtained.

However, model test results can be utilized only if a complete similarity exists between the model and its prototype.

5.6 Similitude – Types of Similarities

"Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties or model and prototype are completely similar."

Following three types of similarities must exist between model and prototype:

1. Geometric Similarity
2. Kinematic Similarity
3. Dynamic Similarity

5.6.1 Geometric Similarity

- The geometric similarity is the similarity of shape. The geometric similarity is said to exist between the model and prototype if the ratio of all corresponding linear dimension in the model and prototype are equal. This ratio is usually known as the scale ratio or scale factor.
- Therefore, geometrically similar objects are similar in their shapes but differ in size as shown in *Fig.5.1*.

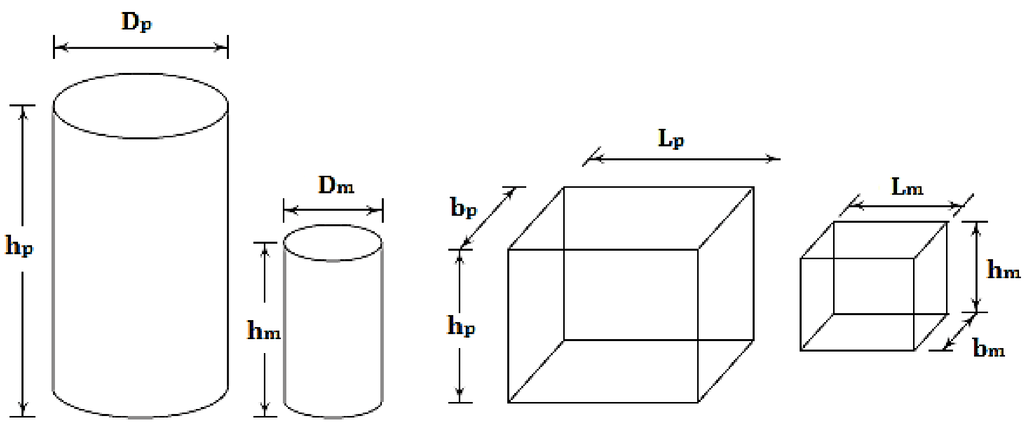

Fig.5.1 – Geometric Similarity

- For geometric similarity,

  Scale Ratio or Scale Factor, $L_r = \frac{L_p}{L_m} = \frac{h_p}{h_m} = \frac{b_p}{b_m}$

5.6.2 Kinematic Similarity

- Kinematic similarity means the ***similarity of motion*** between model and prototype.
- Thus kinematic similarity is said to exist between the model and prototype if the ratio of the velocity and acceleration at the corresponding points in the model & prototype are the same.
- Since velocity and acceleration are vector quantities, it should be the same in magnitude and direction as well.
- A well-known example of kinematic similarity is a planetarium.

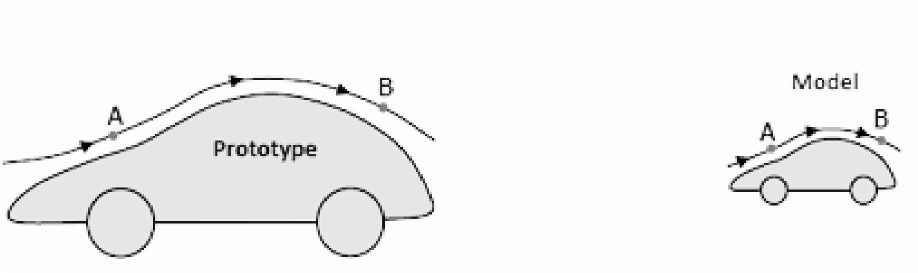

Fig.5.2 – Kinematic Similarity

- For Kinematic similarity,

  Velocity Ratio, $v_r = \frac{v_{p_A}}{v_{m_A}} = \frac{v_{p_B}}{v_{m_B}}$

  Acceleration Ratio, $a_r = \frac{a_{p_A}}{a_{m_A}} = \frac{a_{p_B}}{a_{m_B}}$

The geometric similarity is the necessary condition for the kinematic similarity to be achieved but not a sufficient one.

5.6.3 Dynamic Similarity

- Dynamic similarity means the ***similarity of forces*** between the model and prototype.
- Thus dynamic similarity is said to exist between model and prototype if the ratios of corresponding forces acting at the corresponding points are equal.
- Also, the direction of corresponding forces at the corresponding points should be the same.
- For Dynamic similarity,

  Force Ratio, $F_r = \frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m}$

- In other words, the ratio of magnitudes of any two forces in the prototype must be the same as the ratio of the magnitude of the corresponding forces in the model or the dimensionless numbers discussed earlier should be the same for model and prototype.

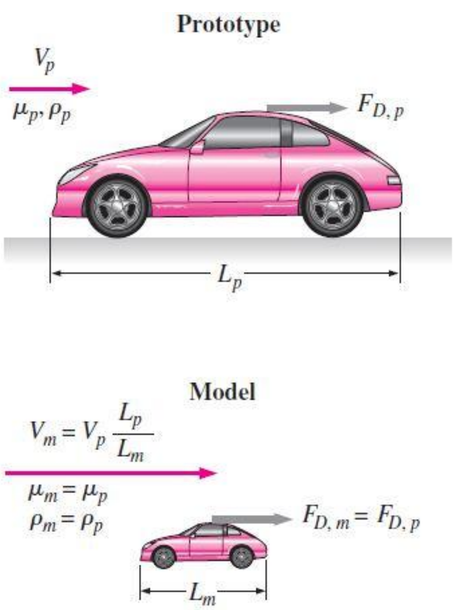

Fig.5.3 – Dynamic Similarity

Both geometric and kinematic similarities are prerequisites for dynamic similarity.

5.7 Model Laws

For dynamic similarity between model and prototype, the dimensionless numbers should be the same for model and prototype. But, It is difficult to satisfy the condition that all dimensionless numbers are the same for model & prototype. Hence models are designed on the basis of the ratio of the force, which is dominating the phenomenon.

"The laws on which the models are designed for dynamic similarity are called Model laws or Similarity laws."

The model laws are:

1. Reynolds Model Law
2. Froude Model Law
3. Euler Model Law
4. Weber Model Law
5. Mach Model Law

5.7.1 Reynolds Model Law

Fluid flow problems where **viscous forces** alone are predominant, the models are designed for dynamic similarity based on **Reynolds** model law.

"It stats that the Reynolds number for the model must be equal to the Reynolds number for the prototype"

According to Reynolds model law,

$$Re_m = Re_p$$

$$\therefore \frac{\rho_m v_m l_m}{\mu_m} = \frac{\rho_p v_p l_p}{\mu_p}$$

$$\therefore \frac{\rho_m v_m l_m}{\rho_p v_p l_p} \times \frac{1}{\frac{\mu_m}{\mu_p}} = 1$$

$$\therefore \boldsymbol{\frac{\rho_r v_r l_r}{\mu_r} = 1}$$

<u>Application of Reynolds model law</u>

- The flow of incompressible fluid in a closed pipe
- Resistance experienced by submarines, aeroplanes, fully immersed bodies, etc.
- Flow-through Venturi meter, Orifice meter, etc.

5.7.2 Froude Model Law

Fluid flow problems where **gravitational forces** alone are predominant, the models are designed for dynamic similarity based on **Froude** model law.

"It stats that the Froude number for the model must be equal to the Froude number for the prototype"

According to Froude model law,

$$Fr_m = Fr_p$$

$$\therefore \frac{v_m}{\sqrt{g_m l_m}} = \frac{v_p}{\sqrt{g_p l_p}}$$

$$\therefore \frac{v_m}{v_p} = \sqrt{\frac{l_m}{l_p}}$$

$$\therefore \boldsymbol{v_r} = \sqrt{\boldsymbol{l_r}}$$

Application of Froude model law

- Free surface flows such as open channel flow, Spillways, weirs, notches, etc.
- The flow of jet from an orifice or nozzle
- Where waves are likely to be formed on the surface
- Where fluids of different densities flow over one another

By using Froude model law, we can derive the following scale ratios (for derivation refer FMHM by R. K. Bansal):

- Scale ratio for time, $\boldsymbol{t_r} = \sqrt{\boldsymbol{l_r}}$
- Scale ratio for acceleration, $\boldsymbol{a_r} = \mathbf{1}$
- Scale ratio for discharge, $\boldsymbol{Q_r} = \boldsymbol{l_r^{2.5}}$
- Scale ratio for force, $\boldsymbol{F_r} = \boldsymbol{l_r^{3}}$
- Scale ratio for pressure, $\boldsymbol{p_r} = \boldsymbol{l_r}$
- Scale ratio for work, energy, torque & moment, $\boldsymbol{T_r} = \boldsymbol{l_r^{4}}$
- Scale ratio for power, $\boldsymbol{P_r} = \boldsymbol{l_r^{3.5}}$

5.7.3 Euler Model Law

Fluid flow problems where **pressure forces** alone are predominant, the models are designed for dynamic similarity based on **Euler** model law.

"It stats that the Euler number for the model must be equal to the Euler number for the prototype"

According to Euler model law,

$$Eu_m = Eu_p$$

$$\therefore \frac{\boldsymbol{v_m}}{\sqrt{\boldsymbol{p_m}/\boldsymbol{\rho_m}}} = \frac{\boldsymbol{v_p}}{\sqrt{\boldsymbol{p_p}/\boldsymbol{\rho_p}}}$$

Application of Euler model law

- Enclosed fluid flow where the turbulence is fully developed (viscous force are negligible, also gravity and surface tension forces are absent).
- Where the phenomenon of cavitation takes place.
- To avoid water hammer effect.

5.7.4 Weber Model Law

Fluid flow problems where **surface tension effects** are predominant, the models are designed for dynamic similarity based on the **Weber** model law.

"It stats that the Weber number for the model must be equal to the Weber number for the prototype"

According to the Weber model law,

$$We_m = We_p$$

$$\therefore \frac{v_m}{\sqrt{\sigma_m/\rho_m l_m}} = \frac{v_p}{\sqrt{\sigma_p/\rho_p l_p}}$$

Application of the Weber model law

- The capillary rise in a narrow passage.
- Capillary movement of water in the soil.
- A thin sheet of liquid flows over a surface.
- Study of droplets and very small jets.

5.7.5 Mach Model Law

Fluid flow problems where **forces due to elastic compression** are predominant, the models are designed for dynamic similarity based on **Mach** model law.

"It stats that the Mach number for the model must be equal to the Mach number for the prototype"

According to Mach model law,

$$M_m = M_p$$

$$\therefore \frac{v_m}{\sqrt{K_m/\rho_m}} = \frac{v_p}{\sqrt{K_p/\rho_p l_p}}$$

Application of Mach model law

- Aerodynamic testing.
- Water hammer problems.
- The flow of air on the aeroplane and projectiles with supersonic speed.
- Underwater testing of torpedoes

5.8 Type of Models

The hydraulic models are classified as :

1. Undistorted models and
2. Distorted models

5.8.1 Undistorted Models

- The models which are geometrically similar to their prototypes or in other words if the scale ratio for the linear dimensions of the model and its prototype is same, model is called **undistorted model.**
- The behavior of the prototype can be easily predicted from the result of undistorted model.

Advantages of undistorted models:

- The behaviour of the prototype can be easily predicted from the result of the undistorted model.
- Same scale ratio is taken for the horizontal and vertical dimension.

Limitations of undistorted models:

- Small dimension such as the height of the model cannot be accurately measured as scale ratio is the same.

5.8.2 Distorted Models

- A model is said to be distorted if it is not geometrically similar to its prototype.
- For a distorted model, different scale ratios for the linear dimensions are adopted.
- For example,

  In the case of rivers, reservoirs etc., two different scale ratios, one for horizontal and other for vertical dimensions are taken. Thus the model of river and reservoir will become a distorted model.

  If for the river, the horizontal and vertical scale ratios are taken to be same so that the model is undistorted model, then the depth of water in a model of the river will be very-very small which may not be measured accurately.

Advantages of the distorted model:

- The vertical dimension of the model can be measured accurately.
- The cost of the model can be reduced.
- Turbulent flow in the model can be maintained.

Limitations of distorted models:

- The results of the distorted model cannot be directly applied to its prototype.

5.9 References

1) Donald F. Elger, Barbara A. Lebret, Clayton T. Crowe & John A. Roberson "Engineering Fluid Mechanics", 11th Edition, 2016, John Wiley & Sons.
2) Yunus A. Cengel & John M. Simbala, "Fluid Mechanics: Fundamentals & Applications", 4th Edition, 2017, McGraw-Hill Education (India) Pvt. Ltd.
3) D. S. Kumar, "Fluid Mechanics & Fluid Power Engineering", S. K. Kataria & Sons.
4) R. K. Bansal, "Fluid Mechanics & Hydraulic Machines", 3rd Edition, 2007, Laxmi Publications (P) Ltd.
5) S. K. Som & G. Biswas, "Introduction to Fluid Mechanics & Fluid Machines", Revised 2nd Edition, 2008, McGraw-Hill Education (India) Pvt. Ltd