7

Flow Through Pipes

Contents

7.1 Introduction 7.2

7.2 Loss of Energy in Pipes 7.2

7.3 Major Energy Losses 7.2

7.4 Minor Energy Losses 7.3

7.1 Introduction

- A fluid is conveyed through close passage when it is required to maintain a certain pressure with respect to atmospheric pressure. Circular pipes, in most of the cases are widely used to carry fluids over a certain distances. They are used to convey water distribution networks, oil transportation, gases for commercial and domestic requirements, air for pneumatic systems and also to transmit power when high pressure fluids are transmitted from one point to another. In hydropower plants large pipes called penstocks are used to transfer high-energy water from reservoir to turbine house.
- Since the fluid in a pipe is in motion, it has to overcome the frictional resistance between the adjacent fluid layers and that between the fluid layer and pipe walls. As fluid flows from one point to another, there is a loss of head due to friction. Thus there is a drop in energy gradient line. In a fully developed pipe flow, the pressure drops linearly along the length of the pipe. Therefore the pressure gradient along the flow remains constant.

7.2 Loss of Energy in Pipes

- When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as:

Energy Losses

Major Energy Losses

This is due to friction and calculated by the following formula:

a) Darcy-Weisbach Formula

b) Chezy's Formula

Minor Energy Losses

This is due to

(a) Sudden expansion of pipe

(b) Sudden contraction of pipe

(c) Bend in pipe

(d) Pipe fittings etc.

(e) An obstruction in pipe.

7.3 Major Energy Losses

- Major energy losses are loss of energy (head) due to friction.

7.3.1 Darcy-Weisbach Formula

- The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in turbulent flow and is given by:

$$h_f = \frac{4fLV^2}{2gd} \qquad \text{Eq. (7.1)}$$

where

$h_f = loss\ of\ head\ due\ to\ friction$

$f = coefficient\ of\ friction\ which\ is\ a\ function\ of\ Reynolds\ number$

$$= \frac{16}{R_e} \; for \; R_e < 2000 \; (viscous \; flow)$$

$$= \frac{0.079}{{R_e}^{1/4}} \; for \; R_e \; varying \; from \; 4000 \; to \; 10^6$$

$L = length \; of \; pipe$

$V = mean \; velocity \; of \; flow$

$d = diameter \; of \; pipe$

7.3.2 Chezy's formula for loss of head due to friction in pipes

- Refer to chapter 6 article 6.5.2 in which expression for loss of head due to friction in pipes is derived. In that section you had came across the following equation,

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad \textit{Eq. (7.2)}$$

where

$h_f = loss \; of \; head \; due \; to \; friction$

$A = area \; of \; cross \; section \; of \; pipe$

$P = wetted \; perimeter \; of \; pipe$

$L = length \; of \; pipe$

$V = mean \; velocity \; of \; flow$

- The ratio of (A/P) is called hydraulic mean depth or hydraulic radius and is denoted by m.

$$\therefore Hydraulic \; mean \; depth, m = \frac{A}{P} = \frac{\frac{\pi}{4} d^2}{\pi d} = \frac{d}{4}$$

- Substituting $\frac{A}{p} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (11.2)

$$h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times V^2$$

$$\therefore V^2 = \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\therefore V = \sqrt{\frac{\rho g}{f'}} \times \sqrt{m \frac{h_f}{L}} \qquad \textit{Eq. (7.3)}$$

- Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant and $\frac{h_f}{L} = i$, where *i* is loss of head per unit length of pipe. Substituting these values in equation (11.3), we got

$$V = C\sqrt{mi} \qquad \textit{Eq. (7.4)}$$

- Equation (7.4) is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to d/4.

7.4 Minor Energy Losses

- The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases :
  1. Loss of head due to sudden enlargement

2. Loss of head due to sudden contraction
3. Loss of head at the entrance of a pipe
4. Loss of head at the exit of a pipe
5. Loss of head due to an obstruction in a pipe
6. Loss of head due to bend in the pipe
7. Loss of head in various pipe fittings

- In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

7.4.1 Loss of head due to Sudden Enlargement

- Consider a liquid flowing through a pipe which has sudden enlargement as shown in *Fig.7.1*. Consider two sections 1-1 and 2-2 before and after the enlargement.
  Let
  p_1 = pressure intensity at section 1-1
  V_1 = celocity of flow at section 1-1
  A_1 = are of pipe at section 1-1
  p_2, V_2 and A_2 = corresponding values at section 2-2.
- Due to sudden change of diameter of the pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary.

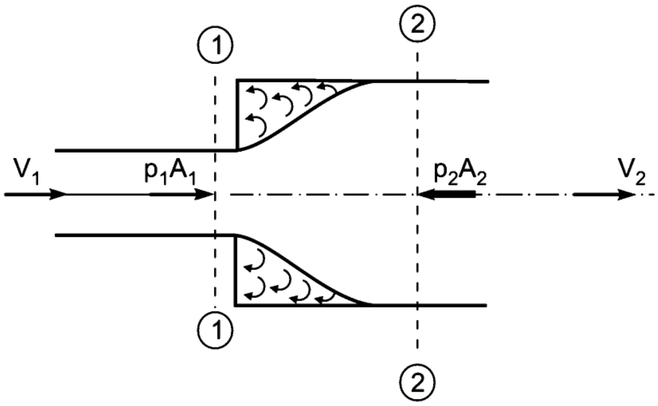

Fig.7.1 – Sudden Enlargement

- Thus the flow separates from the boundary and turbulent eddies are formed as shown in *Fig.7.1*. The loss of head (or energy) takes place due to the formation of these eddies.
- Let
  p' = pressure intensity of the liquid eddies on the area $(A_2 - A_1)$
  h_e = loss of head due to sudden enlargement
- Applying Bernoulli's equation at sections 1-1 and 2-2

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_2+loss\ of\ head\ due\ to\ sudden\ enlargement$$

But $z_1 = z_2$ as the pipe is horizontal

$$\therefore \frac{p_1}{\rho g}+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+h_e$$

$$\therefore h_e=\left(\frac{p_1}{\rho g}-\frac{p_2}{\rho g}\right)+\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right) \qquad Eq.\ (7.5)$$

Consider the control volume of liquid between sections 1-1 and 2-2. Then the force acting on the liquid in the control volume in the direction of flow is given by

$$F_x = p_1A_1 + p'(A_2 - A_1) - p_2A_2$$

But experimentally it is found that $p' = p_1$

$$\therefore F_x = p_1A_1 + p_1(A_2 - A_1) - p_2A_2$$

$$\therefore F_x = (p_1 - p_2)A_2 \qquad Eq.\ (7.6)$$

Momentum of liquid/sec at section 1-1 = mass x velocity

$$= \rho A_1 V_1 \times V_1 = \rho A_1 V_1^2$$

Momentum of liquid/sec at section 2-2 = mass x velocity

$$= \rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$$

$\therefore$ Change of momentum/sec $= \rho A_2 V_2^2 - \rho A_1 V_1^2$

But from comtinuity equation, we have

$$A_1 V_1 = A_2 V_2 \text{ or } A_1 = \frac{A_2 V_2}{V_1}$$

$\therefore$ Change of momentum/sec $= \rho A_2 V_2^2 - \rho \dfrac{A_2 V_2}{V_1} V_1^2 = \rho A_2 V_2^2 - \rho A_2 V_1 V_2$

$$\therefore \text{Change of momentum/sec} = \rho A_2 (V_2^2 - V_1 V_2) \qquad \textit{Eq. (7.7)}$$

Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum or change of momentum per second. Hence equating *Eq. (7.6)* and *Eq. (7.27)*.

$$\therefore (p_1 - p_2) A_2 = \rho A_2 (V_2^2 - V_1 V_2)$$

$$\therefore \frac{(p_1 - p_2)}{\rho} = (V_2^2 - V_1 V_2)$$

Dividing by g on both sides, we have

$$\therefore \frac{(p_1 - p_2)}{\rho g} = \frac{(V_2^2 - V_1 V_2)}{g} \text{ or } \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{(V_2^2 - V_1 V_2)}{g}$$

Substituting $\left(\dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g}\right)$ in equation Eq. (7.5), we get

$$\therefore h_e = \frac{(V_2^2 - V_1 V_2)}{g} + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = \frac{2V_2^2 - 2V_1 V_2 + V_1^2 - V_2^2}{2g}$$

$$\therefore h_e = \frac{V_1^2 - 2V_1 V_2 + V_2^2}{2g}$$

$$\therefore h_e = \frac{(V_1 - V_2)^2}{2g} \qquad \textit{Eq. (7.8)}$$

7.4.2 Loss of head due to sudden contraction

- Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in *Fig.7.2*.
- Consider two sections 1-1 and 2-2 before and after contraction. As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at a section C-C as shown in *Fig.7.2*. This section C-C is called Vena-contracta.
- After section C-C, a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from Vena-contracta to smaller pipe.

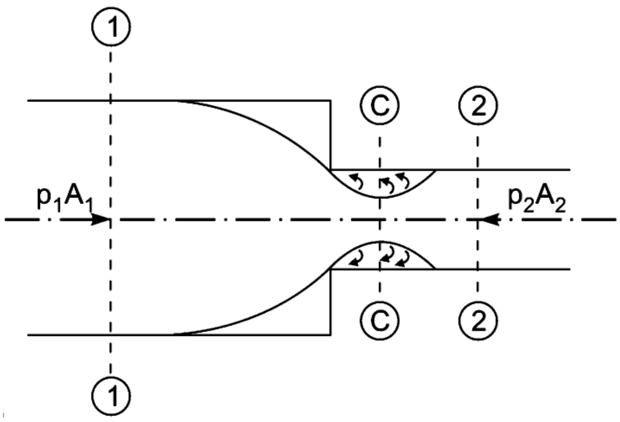

Fig.7.2 – Sudden Contraction

- Let

  A_c = Area of flow at section C-C

  V_c = Velocity of flow at section C-C

  A_2 = Area of flow at section 2-2

  V_2 = Velocity of flow at section 2-2

  h_c = Loss of head due to sudden contraction
- Now h_c = actual loss of head due to enlargement from section C-C to section 2-2 and is given *Eq. (7.27)* as

$$\therefore h_c = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right)^2 \qquad \textit{Eq. (7.9)}$$

- From continuity equation, we have

$$A_c V_c = A_2 V_2 \Rightarrow \frac{\boldsymbol{V_c}}{\boldsymbol{V_2}} = \frac{A_2}{A_c} = \frac{1}{(A_c/A_2)} = \frac{1}{C_c} \qquad \left[\because C_c = \frac{A_c}{A_2}\right]$$

Substituting the value of $\frac{V_c}{V_2}$ in Eq. (7.27), we get

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

$$= \frac{kV_2^2}{2g}, \quad \text{where } k = \left(\frac{1}{C_c} - 1\right)^2$$

- If the value of C_c is assumed to be equal to 0.62, then

$$k = \left(\frac{1}{0.62} - 1\right)^2 = 0.375$$

Then h_c becomes as, $h_c = \frac{kV_2^2}{2g} = 0.375\frac{V_2^2}{2g}$

- If the value of C_c is not given then the head loss due to contraction is taken as

$$h_c = 0.5\frac{V_2^2}{2g} \qquad \textit{Eq. (7.10)}$$

7.4.3 Loss of head at the entrance of a pipe

- This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance.
- In practice the value of loss of head at the entrance (or inlet) of a pipe with sharp cornered entrance is taken as equal to loss of head due to sudden contraction.
- This loss is denoted by h_i

$$h_i = 0.5\frac{V^2}{2g} \qquad \textit{Eq. (7.11)}$$

where V = Velocity of liquid in pipe

7.4.4 Loss of head at the exit of pipe

- This is the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet (if outlet of the pipe is free) or it is lost in the tank or reservoir (if the outlet of the pipe is connected to the tank or reservoir). This loss is equal to $\frac{V^2}{2g}$, where V is the velocity of liquid at the outlet of pipe. This loss is denoted by h_o.

$$h_o = \frac{V^2}{2g} \qquad \textit{Eq. (7.12)}$$

where V = Velocity of liquid in pipe

7.4.5 Loss of head due to an obstruction in a pipe

- Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in *Fig.7.3*.
- Consider a pipe of area of cross-section *A* having an obstruction as shown in *Fig.7.3*.

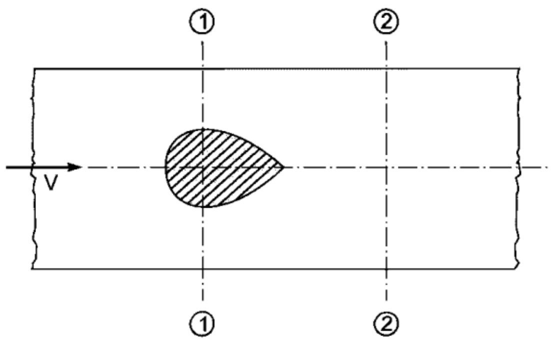

Fig.7.3 – An obstruction in a pipe

- Let

  a = Maximum area of obstruction
  A = Area of pipe
  V = Velocity of liquid in pipe
  (A - a) = Area of flow of liquid at section 1-1
- As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1, after which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to velocity, V in the pipe. This situation is similar to the flow of liquid through sudden enlargement.
- Let, $\boldsymbol{V_c}$ = Velocity of liquid at vena-contracta.
  Then loss of head due to obstruction = loss of head due to enlargement from vena-contracta to section 2-2.

$$= \frac{(V_c - V)^2}{2g} \qquad \text{(i)}$$

From continuity equation,we have $a_c \times V_c = A \times V$ (ii)

where a_c = area of cross-section at vena-contracta
If C_c = co-efficient of contraction, then

$$C_c = \frac{area\ at\ vena - contracta}{(A-a)} = \frac{a_c}{(A-a)}$$

$$\therefore a_c = C_c \times (A-a)$$

Substituting this value in equation (ii), we get

$$C_c \times (A-a) \times V_c = A \times V \Rightarrow V_c = \frac{A \times V}{C_c \times (A-a)}$$

Substituting this value of V_c in equation (i), we get

$$\text{Head loss due to obstruction} = \frac{(V_c - V)^2}{2g} = \frac{\left(\frac{A \times V}{C_c(A-a)} - V\right)^2}{2g} = \frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2 \qquad \textit{Eq. (7.13)}$$

7.4.6 Loss of head due to bend in a pipe

- When there is any bend in a pipe, the velocity of flow changes, due to which the seperation of flow from the boundary and also formation of eddies takes place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as

$$h_b = \frac{kV^2}{2g} \qquad \textit{Eq. (7.14)}$$

where h_b = loss of head due to bend, V = velocity of flow, k = co-efficient of bend

- The vlsue of k depends on
  a) Angle of bend,
  b) Radius of curvature of bend,
  c) Diameter of pipe.

7.4.7 Loss of head in various pipe fittings

- The loss of head in the various pipe fittings such as valves, couplings etc., is expressed as

$$= \frac{kV^2}{2g} \qquad \textit{Eq. (7.15)}$$

where V = velocity of flow, k = co-efficient of pipe fitting.

7.5 Hydraulic Gradient and Total Energy Lines

- The energy gradiet line EGL and the hydraulic gradient line HGL are the graphical representation of the longitudinal variation in total head and piezometric head at salient points of a pipeline.
- The total head with respect to any arbitrary datum is prescribed by the summation of pressure head $(p/\rho g)$, velocity head $(V^2/2g)$, and datum head z.
- Because of friction effects associated with fluid flow and the local resistance arising from pipe transisions and fittings, a part of energy is dissipated. Evidently there is loss of head, and energy drops in the direction of flow by an amount equal to the head loss.
- **Hydraulic Gradient Line (HGL):** It is defined as the line which gives the sum of pressure head $(p/\rho g)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head $(p/\rho g)$ of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as HGL. (Hydraulic Gradient Line).
- **Total Energy Line (TEL) or Energy Gradient Line (EGL):** It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as TEL (Total Energy Line) or Energy Gradient Line (EGL).

7.6 Flow Through Pipes in Series or Flow Through Compound Pipes

- Pipes in series or compound pipes are defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line as shown in *Fig.7.4*.

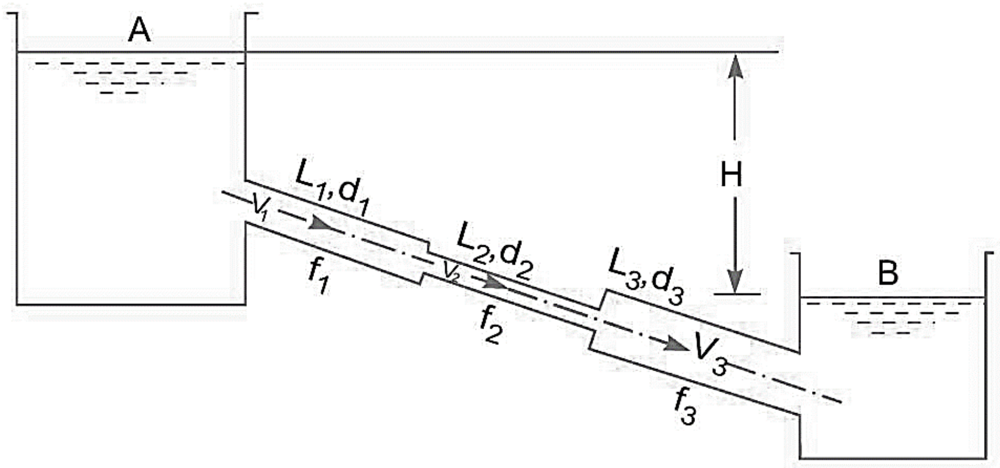

Fig.7.4 – Pipes connected in series

- Let

  L_1, L_2, L_3 = lengths of pipes 1, 2 and 3 respectively
  d_1, d_2, d_3 = diameter of pipes 1, 2 and 3 respectively
  V_1, V_2, V_3 = velocity of flow through pipes 1, 2, 3
  f_1, f_2, f_3 = co-efficient of friction for pipes 1, 2, 3
- As the pipes are in series the discharge passing through each pipe is same.

$$\therefore Q = A_1V_1 = A_2V_2 = A_3V_3$$

- The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$\mathrm{H} = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gd_3} + \frac{V_3^2}{2g} \qquad \textit{Eq. (7.16)}$$

- If minor losses are neglected, then above equation becomes as

$$\mathrm{H} = \frac{4f_1L_1V_1^2}{2gd_1} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3} \qquad \textit{Eq. (7.17)}$$

- If the co-efficient of friction is same for all pipes
  i.e. $f_1 = f_2 = f_3 = f$, then *Eq. (7.27)* becomes as

$$\mathrm{H} = \frac{4fL_1V_1^2}{2gd_1} + \frac{4fL_2V_2^2}{2gd_2} + \frac{4fL_3V_3^2}{2gd_3}$$

$$\mathrm{H} = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right] \qquad \textit{Eq. (7.18)}$$

7.7 Equivalent Pipe

- This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe. The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.
- Let

  L_1 = length of pipe 1 and d_1 = diameter of pipe 1
  L_2 = length of pipe 2 and d_2 = diameter of pipe 2
  L_3 = length of pipe 3 and d_3 = diameter of pipe 3
  H = total head loss

L = length of equivalent pipe

d = diameter of the equivalent pipe

Then L = $L_1 + L_2 + L_3$

- Total head loss in the compound pipe, neglecting minor losses {*Eq. (7.27)*}

$$\mathrm{H} = \frac{4f_1L_1V_1^2}{2gd_1} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3}$$

- Assuming $f_1 = f_2 = f_3 = f$

Discharge,

$$Q = A_1V_1 = A_2V_2 = A_3V_3 = \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2 = \frac{\pi}{4}d_3^2V_3$$

$$\therefore V_1 = \frac{4Q}{\pi d_1^2}, V_2 = \frac{4Q}{\pi d_2^2}, and\ V_3 = \frac{4Q}{\pi d_3^2},$$

- Substituting these values in equation *Eq. (7.27)*, we have

$$\mathrm{H} = \frac{4fL_1\left(\frac{4Q}{\pi d_1^2}\right)^2}{2gd_1} + \frac{4fL_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{2gd_2} + \frac{4fL_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{2gd_3}$$

$$\therefore \mathrm{H} = \frac{4 \times 16fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] \qquad \textit{Eq. (7.19)}$$

- Head loss in the equivalent pipe,

$$\therefore \mathrm{H} = \frac{4fLV^2}{2gd} \text{ [Taking same value of fas in compound pipe]}$$

where $\mathrm{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

$$\therefore \mathrm{H} = \frac{4fL\left(\frac{4Q}{\pi d^2}\right)^2}{2gd} = \frac{4 \times 16fQ^2}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right] \qquad \textit{Eq. (7.20)}$$

- Head loss in compound pipe and in equivalent pipe is same hence equating *Eq. (7.19)* and *Eq. (7.27)*, we have

$$\frac{4 \times 16fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] = \frac{4 \times 16fQ^2}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

$$\therefore \left[\frac{L}{d^5}\right] = \left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] \qquad \textit{Eq. (7.21)}$$

- *Eq. (7.27)* is known as Dupuit's equation. In this equation $L=L_1+L_2+L_3$ and d_1, d_2 and d_3 are known. Hence the equivalent size of the pipe, *i.e.*, value of d can be obtained.

7.8 Flow Through Parallel Pipes

- Consider a main pipe which divides into two or more branches as shown in *Fig.7.5* and again join together downstream to form a single pipe, then the branch pipes are said to be connected in parallel. The discharge through the main is increased by connecting pipes in parallel.

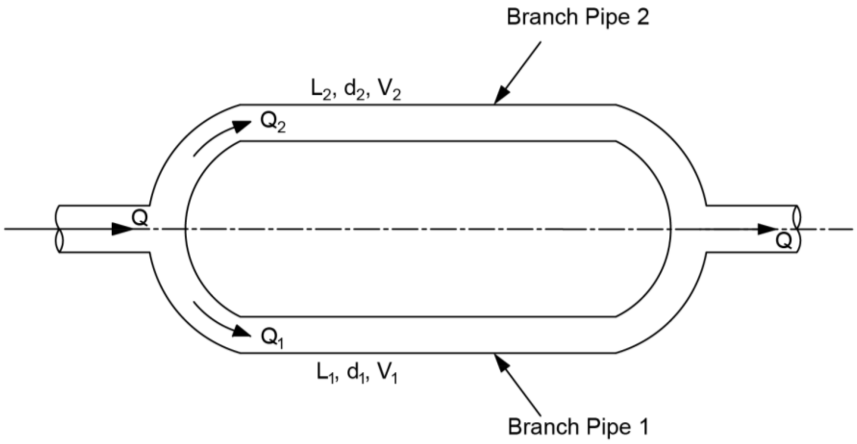

Fig.7.5 – Flow through parallel pipes

- The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes. Hence from *Fig.7.5*, we have

$$Q = Q_1 + Q_2$$

- In this arrangement loss of head in each branch pipe is same

∴ Loss of head for branch pipe 1 = Loss of head for branch pipe 2

$$\frac{4f_1L_1V_1^2}{2gd_1} = \frac{4f_2L_2V_2^2}{2gd_2} \qquad \textit{Eq. (7.22)}$$

- If $f_1 = f_2$, then

$$\frac{L_1V_1^2}{d_1 \times 2g} = \frac{L_2V_2^2}{d_2 \times 2g} \qquad \textit{Eq. (7.23)}$$

7.9 Water Hammer in Pipes

- Consider a long pipe *AB* as shown in *Fig.7.6* connected at one end to a tank containing water at a height of *H* from the centre of the pipe.

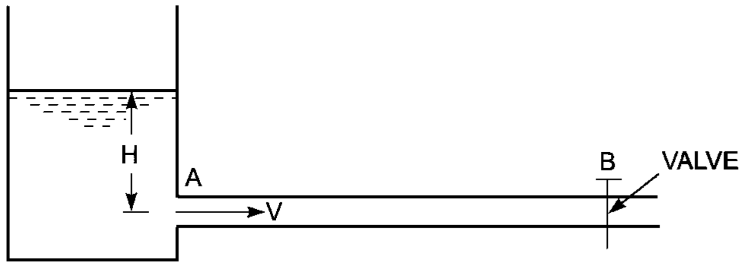

Fig.7.6 – Watter Hammer

- At the other end of the pipe, a valve to regulate the flow of water is provided. When the valve is completely open, the water is flowing with a velocity, *V* in the pipe.
- If now the valve is suddenly closed, the momentum of the flowing water will be destroyed and consequently a wave of high pressure will be set up. This wave of high pressure will be transmitted along the pipe with a velocity equal to the velocity of sound wave and may create noise called knocking. Also this wave of high pressure has the effect of hammering action on the walls of the pipe and hence it is also known as water hammer.
- The pressure rise due to water hammer depends upon :
  - i) the velocity of flow of water in pipe,

ii) the length of pipe,
iii) time taken to close the valve,
iv) elastic properties of the material of the pipe.

- The following cases of water hammer in pipes will be considered:
  1. Gradual closure of valve,
  2. Sudden closure of valve and considering pipe rigid, and
  3. Sudden closure of valve and considering pipe elastic.

7.9.1 Gradual Closure of Valve

- Let the water is flowing through the pipe *AB* shown in *Fig.7.6*, and the valve provided at the end of the pipe is closed gradually.
- Let

A = area of cross-section of the pipe AB,
L = length of pipe,
V = velocity of flow of water through pipe,
T = time in second required to close the valve, and
p = intensity of pressure wave produced.
Mass of water in pipe AB $= \rho \times volume\ of\ water\ = \rho\ \times\ A\ \times\ L$

- The valve is closed gradually in time '*T*' seconds and hence the water is brought from initial velocity V to zero velocity in time T seconds.

$$\therefore Retardation\ of\ water = \frac{Change\ of\ velocity}{Time} = \frac{V-0}{T} = \frac{V}{T}$$

$$\therefore Retarding\ force = mass \times retardation = \rho AL \times \frac{V}{T}$$

- If *p* is the intensity of pressure wave produced due to closure of the valve, the force due to pressure wave,

$$= p \times area\ of\ pipe = p \times A$$

- Equating the above mentioned two forces

$$\rho AL \times \frac{V}{T} = p \times A$$

$$p = \frac{\rho LV}{T} \qquad \textit{Eq. (7.24)}$$

- Head of Pressure

$$H = \frac{p}{\rho g} = \frac{\rho LV}{\rho gT} = \frac{LV}{gT} \qquad \textit{Eq. (7.25)}$$

(i) The valve closure is said to be gradual if $T > \frac{2L}{C}$

(ii) The valve closure is said to be gradual if $T > \frac{2L}{C}$

where T = time in second, and C = velocity of pressure wave.

7.9.2 Sudden Closure of Valve and Pipe is Rigid

- *Eq. (7.24)* gives the relation between increase of pressure due to water hammer in pipe and the time required to close the valve.
- If *t* = 0, the increase in pressure will be infinite. But from experiments, it is observed that the increase in pressure due to water hammer is finite, even for a very rapid closure of valve. Thus *Eq. (7.24)* is valid only for (i) incompressible fluids and (ii) when pipe is rigid.

- But when a wave of high pressure is created, the liquids get compressed to some extent and also pipe material gets stretched. Here we consider a sudden closure of valve [the value of *t* is small and hence a wave of high pressure is created] when pipe is rigid.
- Consider a pipe *AB* in which water is flowing as shown in Fig. 11.32. Let the pipe is rigid and valve fitted at the end *B* is closed suddenly.
- Let

  A = Area of cross-section of pipe *AB*,
  L = Length of pipe,
  V = Velocity of flow of water through pipe,
  p = Intensity of pressure wave produced,
  K = Bulk modulus of water.
- When the valve is closed suddenly, the kinetic energy of the flowing water is converted into strain energy of water if the effect of friction is neglected and pipe wall is assumed perfectly rigid.

$$\therefore Loss\ of\ kinetic\ energy = \frac{1}{2} \times mass\ of\ water \times V^2$$

$$= \frac{1}{2} \times \rho AL \times V^2$$

$$Gain\ of\ strain\ energy = \frac{1}{2}\frac{p^2}{K} \times volume = \frac{1}{2}\frac{p^2}{K} \times AL$$

- Equating loss of kinetic energy to gain of strain energy

$$\therefore \frac{1}{2} \times \rho AL \times V^2 = \frac{1}{2}\frac{p^2}{K} \times AL$$

$$\therefore p^2 = \rho ALV^2 \frac{K}{AL} = \rho KV^2$$

$$\therefore p = \sqrt{\rho KV^2} = V\sqrt{\rho K} = V\sqrt{\frac{\rho^2 K}{\rho}} \qquad Eq.\ (7.26)$$

$$\therefore p = \rho V \times C \qquad \because \sqrt{K/\rho} = C \qquad Eq.\ (7.27)$$

where C = Velocity of sound wave