1

Basics of Stress and Strain

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1.1 Introduction

Materials are classified into elastic, plastic and rigid materials. An elastic material undergoes a deformation when subjected to an external loading such that the deformation disappears on the removal of the loading. A plastic material undergoes a continuous deformation during the period of loading and the deformation is permanent and the material does not regain its original dimensions on the removal of the loading. A rigid material does not undergo any deformation when subjected to an external loading.

In practice no material is absolutely elastic nor plastic nor rigid. We attribute these properties when the deformations are within certain limits. Generally we handle a member in its elastic range. Structural members are all generally designed so as to remain in the elastic condition under the action of the working loads.

1.2 Resistance to Deformation

A material when subjected to an external load system undergoes a deformation. Against this deformation the material will offer a resistance which tends to prevent the deformation. This resistance is offered by the material as long as the member is forced to remain in the deformed condition. This resistance is offered by the material by virtue of its strength.

In the elastic range, the resistance offered by the material is proportional to the deformation brought about on the material by the external loading. The material will have the ability to offer the necessary resistance when the deformation is within a certain limit. A loaded member remains in equilibrium when the resistance offered by the member against the deformation and the applied load are in equilibrium. When the member is incapable of offering the necessary resistance against the external forces, the deformation will continue leading to the failure of the member.

1.3 Stress

The force of resistance per unit area offered by a body against the deformation is called the stress.

$$Stress, \qquad \sigma = \frac{P}{A}$$

Where P = Force acting on a body

A = Cross-sectional area of the body

In S.I. units, the stress is usually expressed in Pascal (Pa), such that 1 Pa = 1 N/m^2.

The external force acting on the body is called the load. The load is applied on the body while the stress is induced in the material of the body.

The following four types of the load are important

1. Dead or steady load: A load is said to be a dead or steady load, when it does not change in magnitude or direction.
2. Live or variable load: A load is said to be a live or variable, when it changes continually.
3. Suddenly applied or shock loads: A load is said to be a suddenly applied or shock load, when it is suddenly applied or removed.
4. Impact load: A load is said to be an impact load, when it is applied with some initial velocity.

1.4 Strain

"When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain."

Strain is dimensionless.

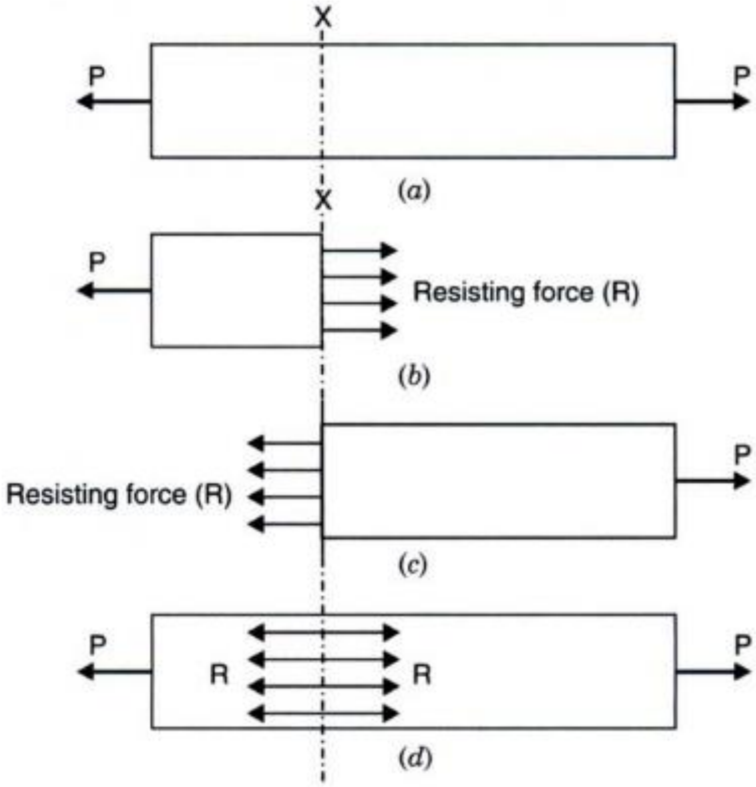


Fig.1.1 – Stress

$$Strain, \qquad \varepsilon = \frac{\delta l}{l}$$

Where δl = Change in length of the body, and

l = Original length of the body.

Tensile Stress and Strain

When a body is subjected to two equal and opposite axial pulls P (also called tensile load) as shown in *Fig.1.9* (a), then the stress induced at any section of the body is known as tensile stress as shown in *Fig.1.9* (b). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as tensile strain.

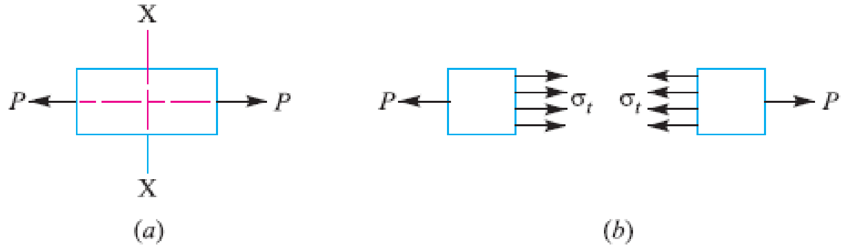


Fig.1.2 – Tensile Stress and Strain

Compressive Stress and Strain

When a body is subjected to two equal and opposite axial pushes P (also called compressive load) as shown in *Fig.1.9* (a), then the stress induced at any section of the body is known as compressive stress as shown in *Fig.1.9* (b). A little consideration will show that due to the compressive load, there will be an increase in cross-sectional area and a decrease in length of the body. The ratio of the decrease in length to the original length is known as compressive strain.

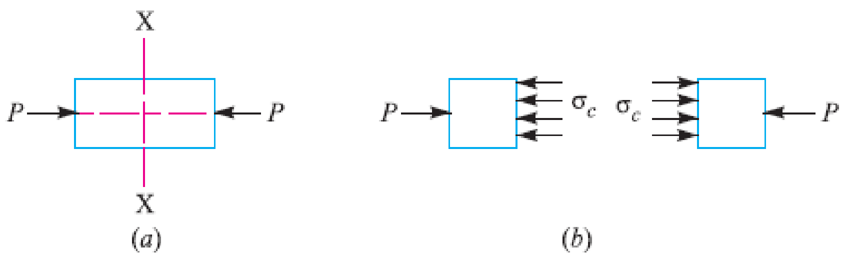


Fig.1.3 – Compressive stress and strain

Young's Modulus or Modulus of Elasticity

Hooke's law states that when a material is loaded within elastic limit, the stress is directly proportional to strain, i.e.

$\sigma \propto \varepsilon$ or $\sigma = E.\varepsilon$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

where E is a constant of proportionality known as *Young's modulus or modulus of elasticity*. In S.I. units, it is usually expressed in GPa i.e. GN/m^2 or kN/mm^2. Hooke's law holds good for tension as well as compression.

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called shear stress.

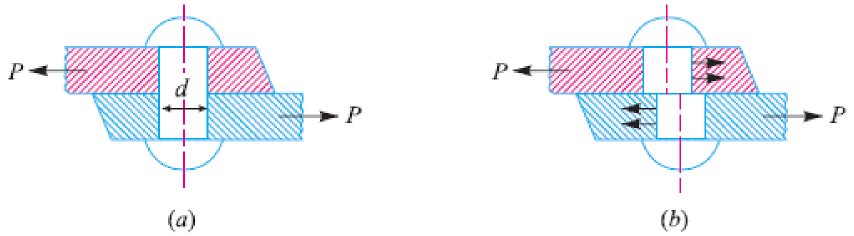


Fig.1.4 – Single shearing of a riveted joint.

The corresponding strain is known as shear strain and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (τ) and phi (ϕ) respectively. Mathematically,

Shear stress, τ = Tangential force / Resisting area

Consider a body consisting of two plates connected by a rivet as shown in *Fig.1.9* (a). In this case, the tangential force P tends to shear off the rivet at one cross-section as shown in *Fig.1.9* (b). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in single shear. In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} d^2$$

And shear stress on the rivet cross-section

$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} d^2} = \frac{4P}{\pi d^2}$$

Now let us consider two plates connected by the two cover plates as shown in *Fig.1.9* (a). In this case, the tangential force P tends to shear off the rivet at two cross-sections as shown in *Fig.1.9* (b). It may be noted that when the tangential force is resisted by two cross-sections of the rivet (or when the shearing takes place at two cross-sections of the rivet), then the rivets are said to be in double shear. In such a case, the area resisting the shear off the rivet,

$$A = 2 \times \frac{\pi}{4} d^2$$

And shear stress on the rivet cross-section

$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} d^2} = \frac{2P}{\pi d^2}$$

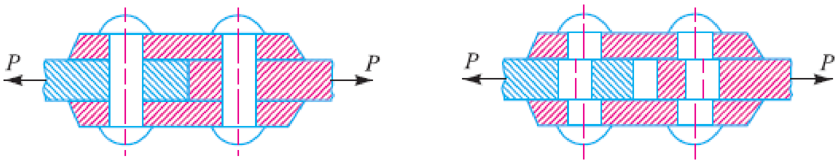


Fig.1.5 – Double shearing of a riveted joint.

Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$\tau \propto \phi$ or $\tau = C.\phi$

where τ = Shear stress,

ϕ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G.

Bearing Pressure

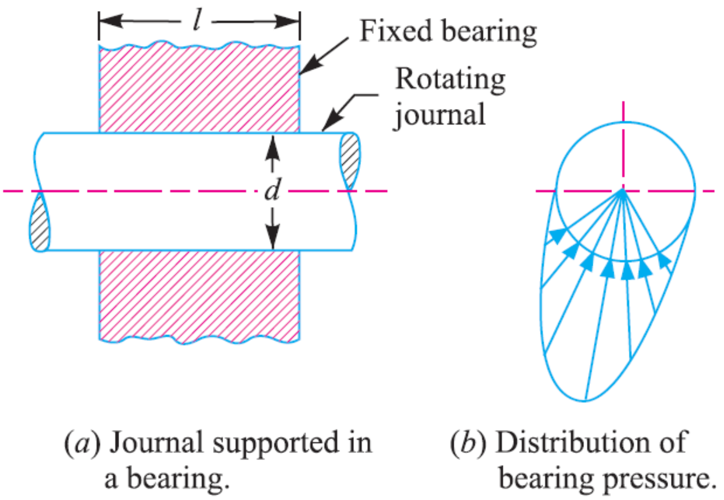


Fig.1.6 – Bearing pressure

The local compression which exists at the surface of contact between two members of a machine part that are in relative motion, is called bearing pressure. This term is commonly used in the design of a journal supported in a bearing, pins for levers, crank pins, clutch lining, etc. Let us consider a journal rotating in a fixed bearing as shown in *Fig.1.9* (a). The journal exerts a bearing pressure on the curved surfaces of the brasses immediately below it. The distribution of this bearing pressure will not be uniform, but it will be in accordance with the shape of the surfaces in contact and deformation characteristics of the two materials.

The distribution of bearing pressure will be similar to that as shown in *Fig.1.9* (b). Since the actual bearing pressure is difficult to determine, therefore the average bearing pressure is usually calculated by dividing the load to the projected area of the curved surfaces in contact. Thus, the average bearing

pressure for a journal supported in a bearing is given by

$$P_b = \frac{P}{l \times d}$$

where pb = Average bearing pressure,

P = Radial load on the journal,

l = Length of the journal in contact, and

d = Diameter of the journal.

Crushing or Bearing Stress

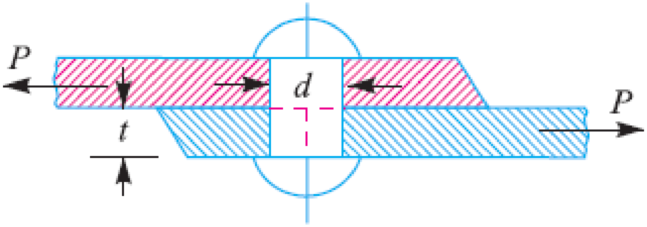


Fig.1.7 – Bearing stress

A localised compressive stress at the surface of contact between two members of a machine part, that are relatively at rest is known as bearing stress or crushing stress. The bearing stress is taken into account in the design of riveted joints, cotter joints, knuckle joints, etc. Let us consider a riveted joint subjected to a load P as shown in *Fig.1.9*. In such a case, the bearing stress or crushing stress (stress at the surface of contact between the rivet and a plate),

$$\sigma_b \ (or \ \sigma_c) = \frac{P}{d \times t \times n}$$

where d = Diameter of the rivet,

t = Thickness of the plate,

d.t = Projected area of the rivet, and

n = Number of rivets per pitch length in bearing or crushing.

Linear or Longitudinal and Lateral Strain

Consider a circular bar of diameter d and length l, subjected to a tensile force P as shown in *Fig.1.9* (a).

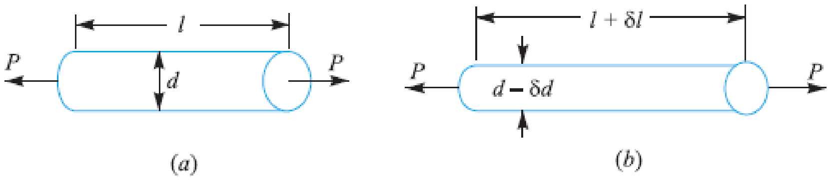


Fig.1.8 – Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δl and the diameter decreases by an amount δd, as shown in *Fig.1.9* (b). Similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as linear strain and an opposite kind of strain in every direction, at right angles to it, is known as lateral strain.

Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain,

Mathematically,

$$\frac{Lateral\ Strain}{Linear\ Strain} = Constant$$

This constant is known as Poisson's ratio and is denoted by 1/m or μ.

Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as volumetric strain.

Mathematically, volumetric strain,

$$\varepsilon_V = \frac{\delta V}{V}$$

where δV = Change in volume, and V = Original volume.

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as bulk modulus. It is usually denoted by K. Mathematically, bulk modulus,

$$K = \frac{Direct\ Stress}{Volumetric\ strain} = \frac{\sigma}{\delta V/V}$$

Relation Between Bulk Modulus and Young's Modulus

The bulk modulus (K) and Young's modulus (E) are related by the following relation,

$$K = \frac{m.E}{3(m-2)} = \frac{E}{3(1-2\mu)}$$

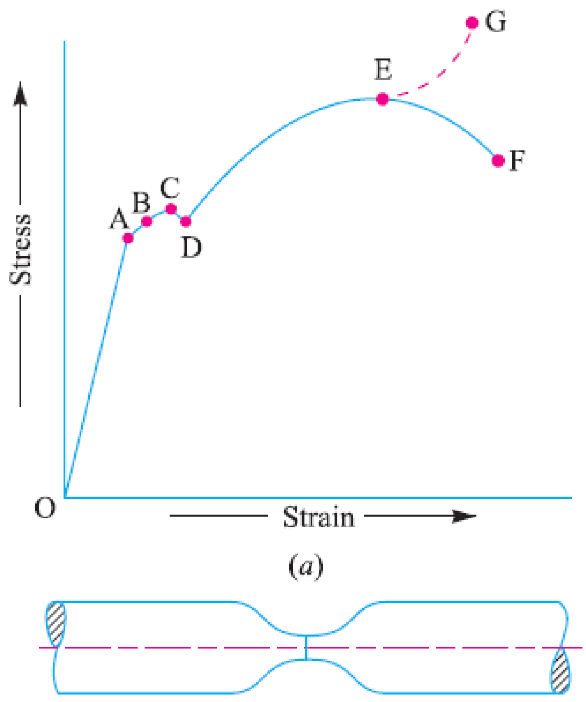
Relation Between Young's Modulus and Modulus of Rigidity

The Young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = \frac{m.E}{2(m+1)} = \frac{E}{2(1+\mu)}$$

1.5 Stress-Strain diagram

In designing various parts of a machine, it is necessary to know how the material will function in service. For this, certain characteristics or properties of the material should be known. The mechanical properties mostly used in mechanical engineering practice are commonly determined from a standard tensile test. This test consists of gradually loading a standard specimen of a material and noting the corresponding values of load and elongation until the specimen fractures. The load is applied and measured by a testing machine. The stress is determined by dividing the load values by the original cross-sectional area of the specimen. The elongation is measured by determining the amounts that two reference points on the specimen are moved apart by the action of the machine. The original distance between the two reference points is known as gauge length. The strain is determined by dividing the elongation values by the gauge length. The values of the stress and corresponding strain are used to draw the stress-strain diagram of the material tested. A stress-strain diagram for a mild steel under tensile test is shown in *Fig.1.9* (a).



(b) Shape of specimen after elongation.

Fig.1.9 – Stress-strain diagram for a mild steel.

The various properties of the material are discussed below:

1. ***Proportional limit.*** We see from the diagram that from point O to A is a straight line, which represents that the stress is proportional to strain. Beyond point A, the curve slightly deviates from the straight line. It is thus obvious, that Hooke's law holds good up to point A and it is known as proportional limit. It is defined as that stress at which the stress-strain curve begins to deviate from the straight line.

2. ***Elastic limit.*** It may be noted that even if the load is increased beyond point A upto the point B, the material will regain its shape and size when the load is removed. This means that the material has elastic properties up to the point B. This point is known as elastic limit. It is defined as the stress developed in the material without any permanent set.

3. ***Yield point.*** If the material is stressed beyond point B, the plastic stage will reach i.e. on the removal of the load, the material will not be able to recover its original size and shape. A little consideration will show that beyond point B, the strain increases at a faster rate with any increase in the stress until the point C is reached. At this point, the material yields before the load and there is an appreciable strain without any increase in stress. In case of mild steel, it will be seen that a small load drops to D, immediately after yielding commences. Hence there are two yield points C and D. The points C and D are called the upper and lower yield points respectively. The stress corresponding to yield point is known as yield point stress.

4. ***Ultimate stress***. At D, the specimen regains some strength and higher values of stresses are required for higher strains, than those between A and D. The stress (or load) goes on increasing till the point E is reached. The gradual increase in the strain (or length) of the specimen is followed with the uniform reduction of its cross-sectional area. The work done, during stretching the specimen, is transformed largely into heat and the specimen becomes hot. At E, the stress, which attains its maximum value is known as ultimate stress. It is defined as the largest stress obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

5. ***Breaking stress***. After the specimen has reached the ultimate stress, a neck is formed, which decreases the cross-sectional area of the specimen, as shown in *Fig.1.9* (b). A little consideration will show that the stress (or load) necessary to break away the specimen, is less than the maximum stress. The stress is, therefore, reduced until the specimen breaks away at point F. The stress corresponding to point F is known as breaking stress.

   The breaking stress (i.e. stress at F which is less than at E) appears to be somewhat misleading. As the formation of a neck takes place at E which reduces the cross-sectional area, it causes the specimen suddenly to fail at F. If for each value of the strain between E and F, the tensile load is divided by the reduced crosssectional area at the narrowest part of the neck, then the true stress-strain curve will follow the dotted line EG.

   However, it is an established practice, to calculate strains on the basis of original cross-sectional area of the specimen.

6. ***Percentage reduction in area***. It is the difference between the original cross-sectional area and cross-sectional area at the neck (i.e. where the fracture takes place). This difference is expressed as percentage of the original cross-sectional area.
   Let A = Original cross-sectional area, and
   a = Cross-sectional area at the neck.
   Then reduction in area = A – a

$$Percentage\ reduction\ in\ area = \frac{A - a}{A} \times 100$$

7. ***Percentage elongation.*** It is the percentage increase in the standard gauge length (i.e. original length) obtained by measuring the fractured specimen after bringing the broken parts together.
   Let l = Gauge length or original length, and
   L = Length of specimen after fracture or final length.
   Elongation = L – l

$$Percentage\ elongation = \frac{L - l}{l} \times 100$$

Ex. 1.1 ***[Ex 4.5; R. S. Khurmi]***

The piston rod of a steam engine is 50 mm in diameter and 600 mm long. The diameter of the piston is 400 mm and the maximum steam pressure is 0.9 N/mm^2. Find the compression of the piston rod if the Young's modulus for the material of the piston rod is 210 kN/mm^2.

Solution:

Given Data:

$d = 50\ mm$

$l = 600\ mm$

$D = 400\ mm$

$p = 0.9\ MPa$

$E = 210\ kN/mm^2$

To be Calculated:

$\delta l = ?$

Cross-sectional area of piston,

$$= \frac{\pi}{4} D^2 = \frac{\pi}{4}(400)^2$$

$$= 125680\ mm^2$$

Maximum load acting on the piston due to steam

$$P = \text{Cross} - \text{sectional area of piston} \times \text{Steam pressure}$$
$$= 125680 \times 0.9 = 113110 \text{ N}$$

Cross-sectional area of piston rod

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(50)^2$$
$$A = 1964 \, mm^2$$

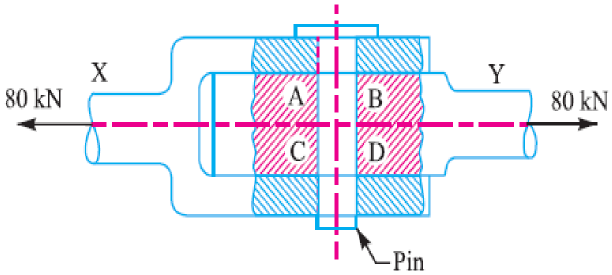
Young's Modulus,

$$E = \frac{P \times l}{A \times \delta l}$$
$$210 \times 10^3 = \frac{113110 \times 600}{1964 \times \delta l}$$
$$\therefore \boldsymbol{\delta l = 0.165 \text{ mm}}$$

Ex. 1.2 ***[Ex 4.7; R. S. Khurmi]***

A pull of 80 kN is transmitted from a bar X to the bar Y through a pin as shown in below Fig. If the maximum permissible tensile stress in the bars is 100 N/mm^2 and the permissible shear stress in the pin is 80 N/mm^2, find the diameter of bars and of the pin.

Solution:



Given Data:

$P = 80 \, kN$

$\sigma_t = 100 \, MPa$

$\tau = 80 \, MPa$

To be Calculated:

a) $D_b = ?$

b) $d_p = ?$

- **Diameter of bars:**

Area of bars,

$$A_b = \frac{\pi}{4}(D_b)^2 = 0.7854 \, (D_b)^2$$

Permissible tensile stress in the bar

$$\sigma_t = \frac{P}{A_b} = \frac{80 \times 10^3}{0.7854 \, (D_b)^2}$$
$$100 = \frac{101846}{(D_b)^2}$$
$$D_b = 32 \, mm$$

- **Diameter of pin:**

Since the tensile load P tends to shear off the pin at two sections i.e. at AB and CD, therefore the pin is in double shear.

Resisting area,

$$A_p = 2 \times \frac{\pi}{4}(D_p)^2 = 1.571 \, (D_p)^2$$

Permissible shear stress in the pin

$$\tau = \frac{P}{A_p} = \frac{80 \times 10^3}{1.571\ (D_p)^2}$$

$$80 = \frac{50.9 \times 10^3}{(D_p)^2}$$

$$D_p = 25.2\ mm$$

1.6 References

1) A Textbook of Machine Design by R.S. Khurmi, S. Chand Publication.
2) Strength of Materials by R.S. Khurmi, S. Chand Publication.
3) Design of Machine Elements by V.B. Bhandari, McGraw-Hill Publication.