4

Torsion

Contents

4.1 Introduction

In workshops and factories, a turning force is always applied to transmit energy by rotation. This turning force is applied either to the rim of a pulley, keyed to the shaft or at any other suitable point at some distance from the axis of the shaft. The product of this turning force and the distance between the point of application of the force and the axis of the shaft is known as torque, turning moment or twisting moment. And the shaft is said to be subjected to torsion. Due to this torque, every cross-section of the shaft is subjected to some shear stress.

4.2 Assumptions for Shear Stress in a Circular Shaft Subjected to Torsion

Following assumptions are made, while finding out shear stress in a circular shaft subjected to torsion:

1. The material of the shaft is uniform throughout.
2. The twist along the shaft is uniform.
3. Normal cross-sections of the shaft, which were plane and circular before the twist, remain plane and circular even after the twist.
4. All diameters of the normal cross-section, which were straight before the twist, remain straight with their magnitude unchanged, after the twist.

A little consideration will show that the above assumptions are justified, if the torque applied is small and the angle of twist is also small.

4.3 Torsional Stresses and Strains

The following assumptions are made in the theory of simple bending:

1. The material of the beam is perfectly homogeneous (i.e., of the same kind throughout) and isotropic (i.e., of equal elastic properties in all directions).
2. The beam material is stressed within its elastic limit and thus, obeys Hooke's law.
3. The transverse sections, which were plane before bending, remains plane after bending also.
4. Each layer of the beam is free to expand or contract, independently, of the layer above or below it.
5. The value of E (Young's modulus of elasticity) is the same in tension and compression.
6. The beam is in equilibrium i.e., there is no resultant pull or push in the beam section.

4.4 Theory of Simple Bending

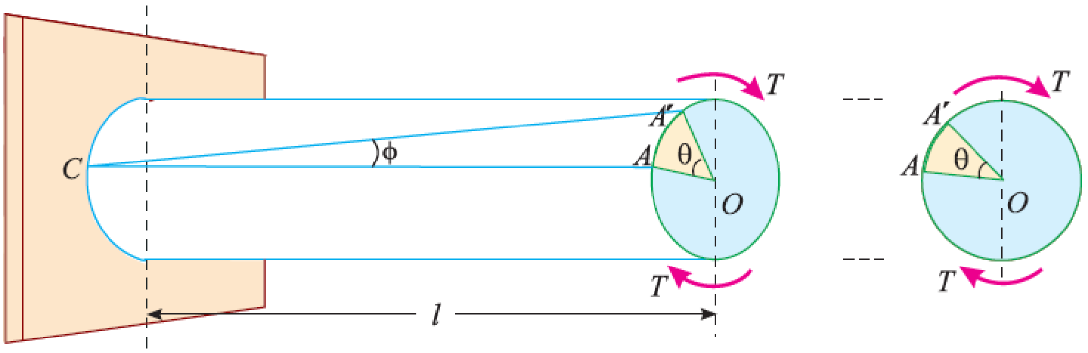

Fig.4.1

Consider a circular shaft fixed at one end and subjected to a torque at the other end as shown in *Fig.4.1*.

Let T = Torque in N-mm,

l = Length of the shaft in mm and

R = Radius of the circular shaft in mm.

As a result of this torque, every cross-section of the shaft will be subjected to shear stresses. Let the line CA on the surface of the shaft be deformed to CA′ and OA to OA′ as shown in *Fig.4.1*.

Let $\angle ACA' = \phi$ in degrees

$\angle AOA' = \theta$ in radians

τ = Shear stress induced at the surface and

C = Modulus of rigidity, also known as torsional rigidity of the shaft material.

We know that shear strain = Deformation per unit length

$$= \frac{AA'}{l} = tan\theta$$

$= \phi$ ...(ϕ being very small, $\tan \phi = \phi$)

We also know that the arc AA′ = R· θ

Eq. (4.1)

$$\phi = \frac{AA'}{l} = \frac{R\theta}{l}$$

If τ is the intensity of shear stress on the outermost layer and C the modulus of rigidity of the shaft, then

Eq. (4.2)

$$\phi = \frac{\tau}{C}$$

From *Eq. (4.1)* and *Eq. (4.2)*, we find that

$$\frac{\tau}{C} = \frac{R\theta}{l}$$

If τ_X be the intensity of shear stress, on any layer at a distance x from the centre of the shaft, then

Eq. (4.3)

$$\frac{\tau_X}{x} = \frac{\tau}{R} = \frac{C.\theta}{l}$$

4.5 Strength of a Solid Shaft

The term, strength of a shaft means the maximum torque or power, it can transmit. As a matter of fact, we are always interested in calculating the torque, a shaft can withstand or transmit.

Let R = Radius of the shaft in mm and

τ = Shear stress developed in the outermost layer of the shaft in N/mm^2

Consider a shaft subjected to a torque T as shown in *Fig.4.2*. Now let us consider an element of area *da* of thickness *dx* at a distance *x* from the centre of the shaft as shown in *Fig.4.2*.

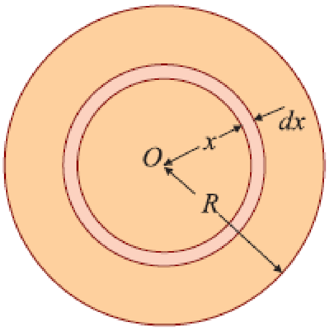

Fig.4.2

$$da = 2\pi x.dx$$

Shear stress at this section,

$$\tau_X = \tau \times \frac{x}{R}$$

where τ = Maximum shear stress.

∴ Turning force = Shear Stress × Area

$$= \tau_X.\ da$$

$$= \tau \times \frac{x}{R} \times da$$

$$= \tau \times \frac{x}{R} \times 2\pi x . dx$$

$$= \frac{2\pi\tau}{R} . x^2 \, dx$$

We know that turning moment of this element,

dT = Turning force × Distance of element from axis of the shaft

$$= \frac{2\pi\tau}{R} . x^2 \, dx . x = \frac{2\pi\tau}{R} . x^3 \, dx$$

The total torque, which the shaft can withstand, may be found out by integrating the above equation between 0 and R i.e.,

$$T = \int_0^R \frac{2\pi\tau}{R} . x^3 \, dx = \frac{2\pi\tau}{R} \int_0^R x^3 \, dx$$

$$T = \frac{2\pi\tau}{R} \left[\frac{x^4}{4} \right]_0^R = \frac{\pi}{2} \tau . R^3$$

$$T = \frac{\pi}{16} \tau . D^3 \quad Nmm$$

where D is the diameter of the shaft and is equal to 2R.

4.6 Strength of a Hollow Shaft

It means the maximum torque or power a hollow shaft can transmit from one pulley to another. Now consider a hollow circular shaft subjected to some torque.

Let R = Outer radius of the shaft in mm,

r = Inner radius of the shaft in mm, and

τ = Maximum shear stress developed in the outer most layer of the shaft material.

Now consider an elementary ring of thickness *dx* at a distance x from the centre as shown in *Fig.4.3*.

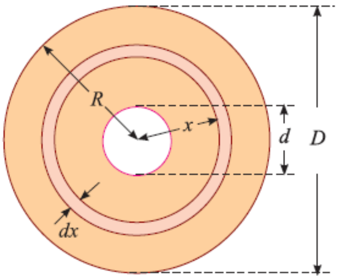

Fig.4.3

We know that area of this ring,

$$da = 2\pi x . dx$$

Shear stress at this section,

$$\tau_X = \tau \times \frac{x}{R}$$

∴ Turning force = Stress × Area

$$= \tau_X . \, da$$

$$= \tau \times \frac{x}{R} \times 2\pi x . dx$$

$$= \frac{2\pi\tau}{R} . x^2 \, dx$$

We know that turning moment of this element,

dT = Turning force × Distance of element from axis of the shaft

$$= \frac{2\pi\tau}{R} . x^2 \, dx . x = \frac{2\pi\tau}{R} . x^3 \, dx$$

The total torque, which the shaft can transmit, may be found out by integrating the above equation between r and R.

$$T = \int_r^R \frac{2\pi\tau}{R}.x^3\,dx = \frac{2\pi\tau}{R}\int_r^R x^3\,dx$$

$$T = \frac{2\pi\tau}{R}\left[\frac{x^4}{4}\right]_r^R = \frac{2\pi\tau}{R}\left(\frac{R^4 - r^4}{4}\right)$$

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right) \qquad N.mm$$

where D is the external diameter of the shaft and is equal to 2R and 2d is the internal diameter of the shaft and is equal to 2r.

4.7 Power Transmitted by a Shaft

The main purpose of a shaft is to transmit power from one shaft to another in factories and workshops. Now consider a rotating shaft, which transmits power from one of its ends to another.

Let N = No. of revolutions per minute and

T = Average torque in kN-m.

Work done per minute = Force × Distance

= T × 2πN

= 2πNT

$$Work\ done\ per\ second = \frac{2\pi NT}{60}\ \ kN.m$$

Power transmitted = Work done in kN-m per second

$$= \frac{2\pi NT}{60}\ \ kW$$

4.8 References

1) A Textbook of Machine Design by R.S. Khurmi, S. Chand Publication.
2) Strength of Materials by R.S. Khurmi, S. Chand Publication.
3) Design of Machine Elements by V.B. Bhandari, McGraw-Hill Publication.