7

Beams and Columns

Contents

7.1 Introduction

- A machine part subjected to an axial compressive force is called a strut.
- A strut may be horizontal, inclined or even vertical. But a vertical strut is known as a column, pillar or stanchion.
- The machine members that must be investigated for column investigated for column action are piston rods, valve push rods, connecting rods, screw jack, side links of toggle jack etc.

7.2 Failure of a Column or Strut

- It has been observed that when a column or strut is subjected to a compressive load and the load is gradually increased, a stage will reach when the column will be subjected to ultimate load. Beyond this, the column will fail by crushing and the load will be known as crushing load.
- It has also been experienced, that sometimes, a compression member does not fail entirely by crushing, but also by bending i.e. buckling. This happens in the case of long columns.
- It has also been observed that all the **short column** (The columns which have lengths less than 8 times their diameter are called short columns) is fail due to their crushing. But, if a **long column** (The columns which have lengths more than 30 times their diameter are called long columns) is subjected to a compressive load, it is subjected to a compressive stress.
- If the load is gradually increased, the column will reach a stage, when it will start buckling. The load, at which the column tends to have lateral displacement or tends to buckle, is called buckling load, critical load, or crippling load and the column is said to have developed an elastic instability.
- The buckling takes place about the axis having minimum radius of gyration or least moment of inertia. It may be noted that for a long column, the value of buckling load will be less than the crushing load. Moreover, the value of buckling load is low for long columns, and relatively high for short columns.

7.3 Types of End Conditions of Columns

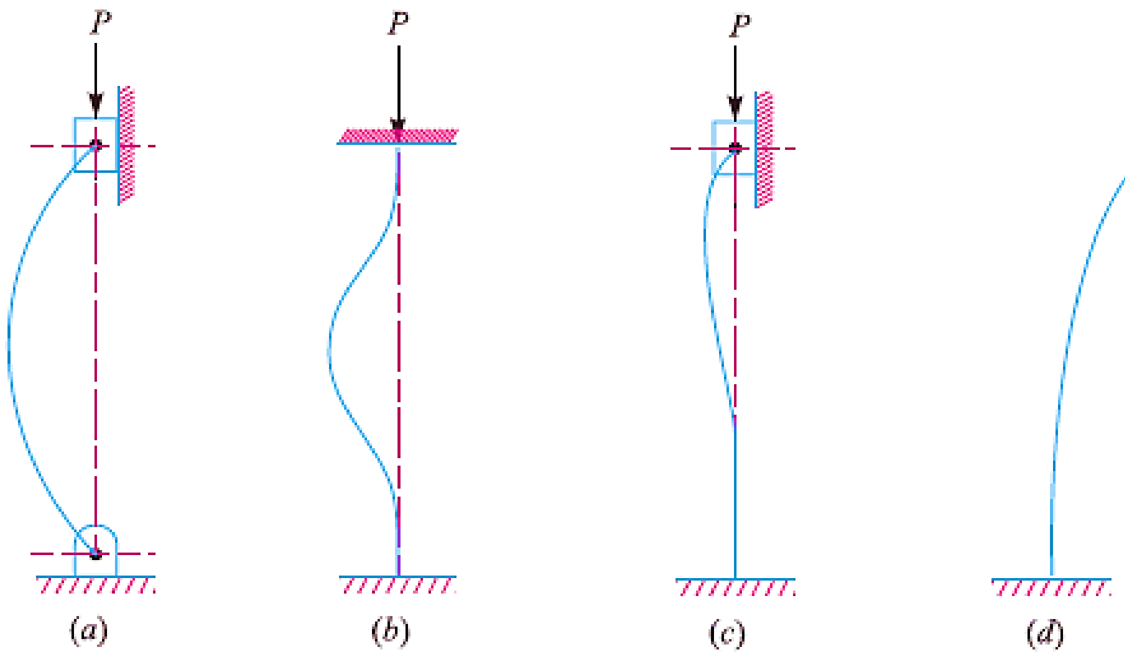


Fig.7.1 – Types of end conditions of columns

- In actual practice, there are a number of end conditions for columns. But we shall study the Euler's column theory on the following four types of end conditions which are important from the subject point of view:
  1. Both the ends hinged or pin jointed as shown in *Fig.7.7*(a),
  2. Both the ends fixed as shown in *Fig.7.7*(b),
  3. One end is fixed and the other hinged as shown in *Fig.7.7*(c), and
  4. One end is fixed and the other free as shown in *Fig.7.7*(d).

7.4 Euler's Column Theory

- The first rational attempt, to study the stability of long columns, was made by Mr. Euler. He derived an equation, for the buckling load of long columns based on the bending stress.
- While deriving this equation, the effect of direct stress is neglected. This may be justified with the statement, that the direct stress induced in a long column is negligible as compared to the bending stress.
- It may be noted that Euler's formula cannot be used in the case of short columns, because the direct stress is considerable, and hence cannot be neglected.

7.4.1 Assumptions in Euler's Column Theory

The following simplifying assumptions are made in Euler's column theory:

1. Initially the column is perfectly straight, and the load applied is truly axial.
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic, and thus obeys Hooke's law.
4. The length of column is very large as compared to its cross-sectional dimensions.
5. The shortening of column, due to direct compression (being very small) is neglected.
6. The failure of column occurs due to buckling alone.
7. The weight of the column itself is neglected.

7.4.2 Euler's Formula

According to Euler's theory, the crippling or buckling load (Wcr) under various end conditions is represented by a general equation,

$$W_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{C\pi^2 E\,Ak^2}{l^2} \qquad (I = Ak^2)$$

$$= \frac{C\pi^2 EA}{\left({}^{l}/_{k}\right)^2}$$

where E = Modulus of elasticity or Young's modulus for the material of the column,

A = Area of cross – section,

k = Least radius of gyration of the cross – section,

l = Length of the column, and

C = Constant, representing the end conditions of the column or end fixity coefficient.

The following ***Table 7.1*** shows the values of end fixity coefficient (C) for various end conditions.

Table 7.1 - Values of end fixity coefficient

Sr. No.	End Conditions	End fixity coefficient (C)
1	Both ends hinged	1
2	Both ends fixed	4
3	One end fixed and other hinged	2
4	One end fixed and other end free	0.25

7.4.3 Slenderness Ratio

- In Euler's formula, the ratio l / k is known as slenderness ratio. It may be defined as the ratio of the effective length of the column to the least radius of gyration of the section.
- It may be noted that the formula for crippling load, in the previous article is based on the assumption that the slenderness ratio l / k is so large, that the failure of the column occurs only due to bending, the effect of direct stress (i.e. W / A) being negligible.

7.4.4 Limitations of Euler's Formula

The general equation for the crippling load is

$$W_{cr} = \frac{C\pi^2 EA}{\left(l/_k\right)^2}$$

Crippling Stress

$$\sigma_{cr} = \frac{W_{cr}}{A} = \frac{C\pi^2 E}{\left(l/_k\right)^2}$$

- A little consideration will show that the crippling stress will be high, when the slenderness ratio is small. We know that the crippling stress for a column cannot be more than the crushing stress of the column material. It is thus obvious that the Euler's formula will give the value of crippling stress of the column (equal to the crushing stress of the column material) corresponding to the slenderness ratio. Now consider a mild steel column. We know that the crushing stress for mild steel is 330 N/mm^2 and Young's modulus for mild steel is 0.21 × 10^6 N/mm^2.
- Now equating the crippling stress to the crushing stress, we have

$$\frac{C\pi^2 E}{\left(l/_k\right)^2} = 330$$

$$\frac{1 \times 9.87 \times 0.21 \times 10^6}{\left(l/_k\right)^2} = 330$$

$$\left(l/_k\right)^2 = 6281$$

$$l/_k = 79.25 \approx 80$$

- Hence if the slenderness ratio is less than 80, Euler's formula for a mild steel column is not valid.
- Sometimes, the columns whose slenderness ratio is more than 80 are known as long columns, and those whose slenderness ratio is less than 80 are known as short columns. It is thus obvious that the Euler's formula holds good only for long columns.

7.4.5 Equivalent Length of a Column

– Sometimes, the crippling load according to Euler's formula may be written as

$$W_{cr} = \frac{\pi^2 E\, I}{L^2}$$

– where L is the equivalent length or effective length of the column. The equivalent length of a given column with given end conditions is the length of an equivalent column of the same material and cross – section with hinged ends to that of the given column.

– The relation between the equivalent length and actual length for the given end conditions is shown in the following ***Table** 7.2*.

Table 7.2 - Relation between equivalent length (L) and actual length (l)

Sr. No.	Mechanical Properties	Defination
1	Both ends hinged	L = l
2	Both ends fixed	$L = {}^{l}/_{2}$
3	One end fixed and other hinged	$L = {}^{l}/_{\sqrt{2}}$
4	One end fixed and other end free	L =2l

7.5 Rankine's Formula for Columns

- We have already discussed that Euler's formula gives correct results only for very long columns. Though this formula is applicable for columns, ranging from very long to short ones, yet it does not give reliable results. Prof. Rankine, after a number of experiments, gave the following empirical formula for columns.

Eq. (7.1)

$$\frac{1}{W_{cr}} = \frac{1}{W_c} + \frac{1}{W_E}$$

where W_{cr} = Crippling load by Rankine's formula,

W_c = Ultimate crushing load for the column $= \sigma_c \times A$,

W_E = Crippling load, obtained by Euler's formula = $\frac{\pi^2 E\, I}{L^2}$

– A little consideration will show that the value of W_C will remain constant irrespective of the fact whether the column is a long one or short one. Moreover, in the case of short columns, the value of W_E will be very high, therefore the value of $1/W_E$ will be quite negligible as compared to $1/W_C$.

– It is thus obvious, that the Rankine's formula will give the value of its crippling load (i.e. W_{cr}) approximately equal to the ultimate crushing load (i.e. W_C).

– In case of long columns, the value of W_E will be very small, therefore the value of $1/W_E$ will be quite considerable as compared to $1/W_C$.

– It is thus obvious, that the Rankine's formula will give the value of its crippling load (i.e. W_{cr}) approximately equal to the crippling load by Euler's formula (i.e. W_E).

– Thus, we see that Rankine's formula gives a fairly correct result for all cases of columns, ranging from short to long columns.

– From *Eq. (7.1)*, we know that

$$\frac{1}{W_{cr}} = \frac{1}{W_c} + \frac{1}{W_E} = \frac{W_E + W_c}{W_c \times W_E}$$

$$W_{cr} = \frac{W_c \times W_E}{W_E + W_c} + \frac{W_c}{1 + \frac{W_c}{W_E}}$$

– Now substituting the value of W_C and W_E in the above equation, we have

$$W_{cr} = \frac{\sigma_c \times A}{1 + \frac{\sigma_c \times A \times L^2}{\pi^2 E\, I}} = \frac{\sigma_c \times A}{1 + \frac{\sigma_c}{\pi^2 E} \times \frac{A.L^2}{A.k^2}} \qquad (I = Ak^2)$$

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k}\right)^2} = \frac{Crushing\ Load}{1 + a\left(\frac{L}{k}\right)^2}$$

where σ_c = Crushing stress or yield stress in compression,

A = Cross – sectional area of the column,

a = Rankine's constant = $\frac{\sigma_c}{\pi^2 E}$

L = Equivalent length of the column, and

k = Least radius of gyration.

– The following ***Table 7.3*** gives the values of crushing stress and Rankine's constant for various materials.

Table 7.3 - Values of crushing stress ($\boldsymbol{\sigma_c}$) and Rankine's constant (a) for various materials.

Sr. No.	Material	σ_c in MPa	$a = \frac{\sigma_c}{\pi^2 E}$
1	Wrought iron	250	$\frac{1}{9000}$
2	Cast iron	550	$\frac{1}{1600}$
3	Mild steel	320	$\frac{1}{7500}$
4	Timber	50	$\frac{1}{750}$

7.6 Johnson's Formulae for Columns

Prof. J.B. Johnson proposed the following two formula for short columns.

1. **Straight line formula:** According to straight line formula proposed by Johnson, the critical or crippling load is

$$W_{cr} = A\left[\sigma_y - \frac{2\sigma_y}{3\pi}\left(\frac{L}{k}\right)\sqrt{\frac{\sigma_y}{3C \times E}}\right] = A\left[\sigma_y - C_1\left(\frac{L}{k}\right)\right]$$

where A = Cross – sectional area of column,

σ_y = Yield point stress,

$$C_1 = \frac{2\sigma_y}{3\pi}\sqrt{\frac{\sigma_y}{3C \times E}}$$

= A constant, whose value depends upon the type of material as well as the type of ends, and

$\frac{L}{k} = Slenderness\ ratio$

– If the safe stress (Wcr / A) is plotted against slenderness ratio (L / k), it works out to be a straight line, so it is known as straight line formula.

2. **Parabolic formula:** Prof. Johnson after proposing the straight line formula found that the results obtained by this formula are very approximate. He then proposed another formula, according to which the critical or crippling load,

$$W_{cr} = A \times \sigma_y \left[1 - \frac{\sigma_y}{4C\pi^2 E}\left(\frac{L}{k}\right)^2\right]$$

– If a curve of safe stress (Wcr / A) is plotted against (L / k), it works out to be a parabolic, so it is known as parabolic formula.

– *Fig.7.7* shows the relationship of safe stress (Wcr / A) and the slenderness ratio (L / k) as given by Johnson's formula and Euler's formula for a column made of mild steel with both ends hinged (i.e. C=1), having a yield strength, σy = 210 MPa. We see from the figure that point A (the point of tangency between the Johnson's straight line formula and Euler's formula) describes the use of two formulae. In other words, Johnson's straight line formula may be used when L / k < 180 and the Euler's formula is used when L / k >180.

– Similarly, the point B (the point of tangency between the Johnson's parabolic formula and Euler's formula) describes the use of two formulae. In other words, Johnson's parabolic formula is used when L / k <140 and the Euler's formula is used when L / k >140.

– Note: For short columns made of ductile materials, the Johnson's parabolic formula is used.

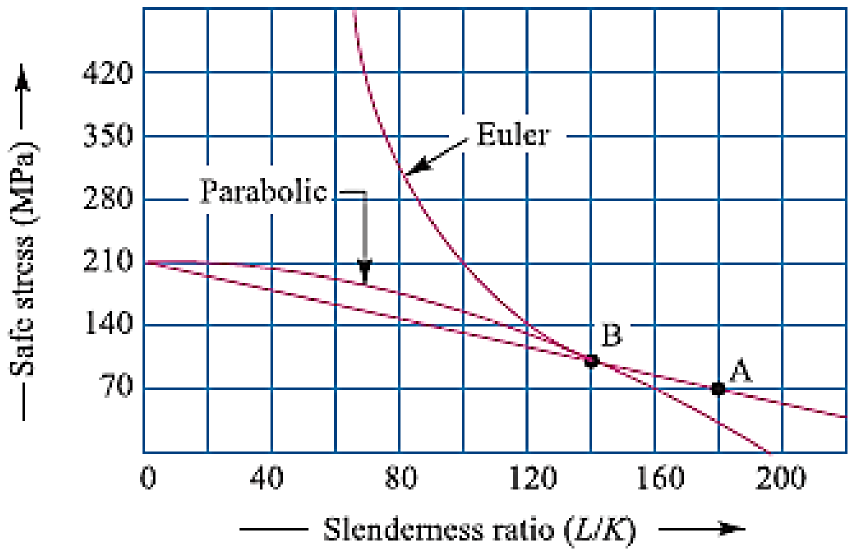


Fig.7.2 – Relation between slenderness ratio and safe stress

7.7 Long Columns Subjected to Eccentric Loading

- In the previous articles, we have discussed the effect of loading on long columns. We have always referred the cases when the load acts axially on the column (i.e. the line of action of the load coincides with the axis of the column). But in actual practice it is not always possible to have an axial load on the column, and eccentric loading takes place. Here we shall discuss the effect of eccentric loading on the Rankine's and Euler's formula for long columns.

- Consider a long column hinged at both ends and subjected to an eccentric load as shown in *Fig.7.7*.

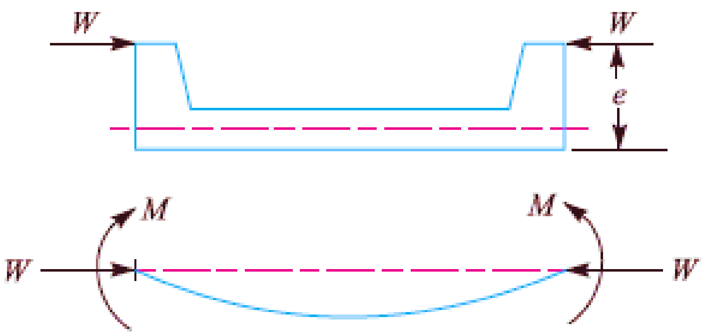


Fig.7.3 – Long column subjected to eccentric loading

Let W = Load on the column,

A = Area of cross – section,

e = Eccentricity of the load,

Z = Section modulus,

yc = Distance of the extreme fibre (on compression side) from the axis of the column,

k = Least radius of gyration,

I = Moment of inertia = A k^2,

E = Young's Modulus, and

l = Length of the column.

- We have already discussed that when a column is subjected to an eccentric load, the maximum intensity of compressive stress is given by the relation

$$\sigma_{max} = \frac{W}{A} + \frac{M}{Z}$$

- The maximum bending moment for a column hinged at both ends and with eccentric loading is given by

$$M = W.e.sec\frac{l}{2}\sqrt{\frac{W}{E.I}} = W.e.sec\frac{l}{2k}\sqrt{\frac{W}{E.A}} \qquad (I = Ak^2)$$

$$\sigma_{max} = \frac{W}{A} + \frac{W.e.sec\frac{l}{2k}\sqrt{\frac{W}{E.A}}}{Z}$$

$$= \frac{W}{A} + \frac{W.e.y_c.sec\frac{l}{2k}\sqrt{\frac{W}{E.A}}}{Ak^2} \qquad \left(Z = {}^{I}/_{y_c} = {}^{Ak^2}/_{y_c}\right)$$

$$= \frac{W}{A} + \left[1 + \frac{e.y_c}{k^2}sec\frac{l}{2k}\sqrt{\frac{W}{E.A}}\right]$$

$$= \frac{W}{A} + \left[1 + \frac{e.y_c}{k^2}sec\frac{L}{2k}\sqrt{\frac{W}{E.A}}\right]$$

…(Substituting l = L, equivalent length for both ends hinged).

7.8 Design of Piston Rod

- Since a piston rod moves forward and backward in the engine cylinder, therefore it is subjected to alternate tensile and compressive forces. It is usually made of mild steel. One end of the piston rod is secured to the piston by means of tapered rod provided with nut. The other end of the piston rod is joined to crosshead by means of a cotter.

  Let p = Pressure acting on the piston,

  D = Diameter of the piston,

  d = Diameter of the piston rod,

  W = Load acting on the piston rod,

  W_{cr} = Buckling or crippling load = W x Factor of safety,

  σ_t = Allowable tensile stress for the material of rod,

  σ_c = Compressive yield stress,

  A = Cross – sectional area of the rod,

  l = Length of the rod, and

  k = Least radius of gyration of the rod section.

The diameter of the piston rod is obtained as discussed below:

1. When the length of the piston rod is small i.e. when slenderness ratio (l / k) is less than 40, then the diameter of piston rod may be obtained by equating the load acting on the piston rod to its tensile strength, i.e.

$$W = \frac{\pi}{4} \times d^2 \times \sigma_t$$

$$\frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} \times d^2 \times \sigma_t$$

$$d = D\sqrt{\frac{p}{\sigma_t}}$$

2. When the length of piston rod is large, then the diameter of the piston rod is obtained by using Euler's formula or Rankine's formula. Since the piston rod is securely fastened to the piston and cross head, therefore it may be considered as fixed ends. The Euler's formula is

$$W_{cr} = \frac{\pi^2 E\, I}{L^2}$$

   and Rankine's formula is,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k}\right)^2}$$

7.9 Design of Push Rods

- The push rods are used in overhead valve and side valve engines. Since these are designed as long columns, therefore Euler's formula should be used. The push rods may be treated as pin end columns because they use spherical seated bearings.

  Let W = Load acting on the push rod,

  D = Diameter of the push rod,

d = Diameter of the hole through the push rod,

I = Moment of inertia of the push rod,

$$= \frac{\pi}{64} D^4, \qquad for\ solid\ rod$$

$$= \frac{\pi}{64} (D^4 - d^4), \qquad for\ tubular\ section$$

l = Length of the push rod, and

E = Young's modulus for the material of push rod.

- If m is the factor of safety for the long columns, then the critical or crippling load on the rod is given by

$$W_{cr} = m \times W$$

- Now using Euler's formula, $W_{cr} = \frac{\pi^2 E\ I}{L^2}$ the diameter of the push rod (D) can be obtained.

Notes:

1. Generally the diameter of the hole through the push rod is 0.8 times the diameter of push rod, i.e. d = 0.8 D
2. Since the push rods are treated as pin end columns, therefore the equivalent length of the rod (L) is equal to the actual length of the rod (l).

7.10 Design of Connecting Rod

- A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore the cross – section of the connecting rod is designed as a strut and the Rankine's formula is used.
- A connecting rod subjected to an axial load W may buckle with X – axis as neutral axis (i.e. in the plane of motion of the connecting rod) or Y – axis as neutral axis (i.e. in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about X – axis and both ends fixed for buckling about Y – axis. A connecting rod should be equally strong in buckling about either axes.

Let A = Cross – sectional area of the connecting rod,

l = Length of the connecting rod,

σ_c = Compressive yield stress,

W_{cr} = Crippling or buckling load,

I_{xx} and I_{yy} = Moment of inertia of the section about X – axis and Y – axis respectively,

k_{xx} and k_{yy} = Radius of gyration of the section about X – axis and Y – axis respectively.

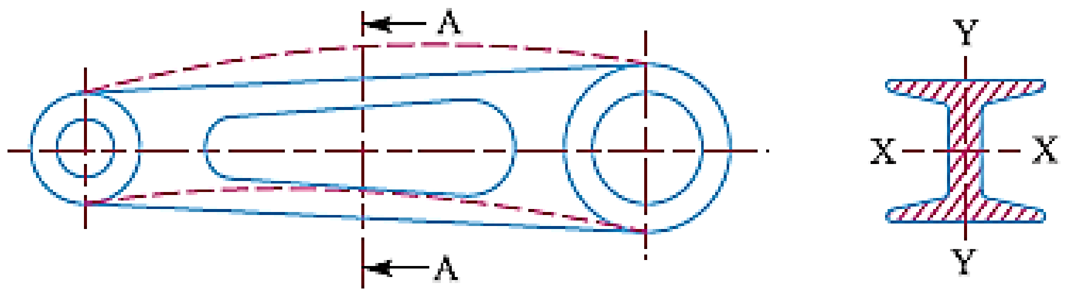


Fig.7.4 – Buckling of connecting rod

According to Rankine's formula,

$$W_{cr}\ about\ X-axis = \frac{\sigma_c \times A}{1+a\left(\frac{L}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1+a\left(\frac{l}{k_{xx}}\right)^2} \qquad (For\ both\ ends\ hinged, L = l)$$

$$W_{cr}\ about\ Y-axis = \frac{\sigma_c \times A}{1+a\left(\frac{L}{k_{yy}}\right)^2} = \frac{\sigma_c \times A}{1+a\left(\frac{l}{2k_{yy}}\right)^2} \qquad (For\ both\ ends\ fixed, L = \frac{l}{2})$$

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal,

$$\frac{\sigma_c \times A}{1+a\left(\frac{l}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1+a\left(\frac{l}{2k_{yy}}\right)^2} \qquad or \qquad \left(\frac{l}{k_{xx}}\right)^2 = \left(\frac{l}{2k_{yy}}\right)^2$$

$$(k_{xx})^2 = 4(k_{yy})^2 \qquad or \qquad I_{xx} = 4.I_{yy} \qquad (I = Ak^2)$$

This shows that the connecting rod is four times strong in buckling about Y – axis than about X- axis. If $I_{xx} > 4\ I_{yy}$, then buckling will occur about Y – axis and $I_{xx} < 4\ I_{yy}$, buckling about X – axis. In actual practice, I_{xx} is kept slightly less than 4 I_{yy}. It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X – axis. The design will always be satisfactory for buckling about Y – axis.

The most suitable section for the connecting rod is I – section with the proportions as shown in Fig. 7.5(a).

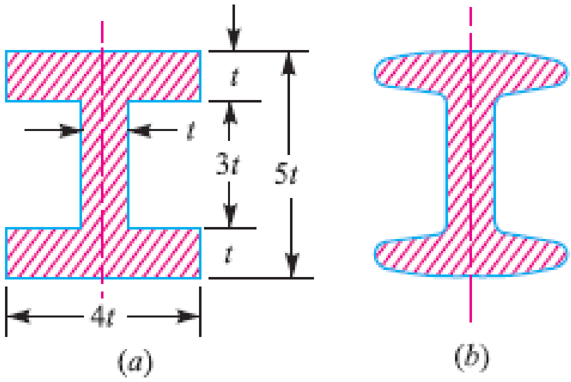


Fig.7.5 I – section of connecting rod

$$Area\ of\ the\ section = 2(4t \times t) + (3t \times t) = 11t^2$$

$$Moment\ of\ inertia\ about\ X-axis, I_{xx} = \frac{1}{12}[4t(5t)^3 - 3t(3t)^3] = \frac{419}{12}t^4$$

$$Moment\ of\ inertia\ about\ Y-axis, I_{yy} = \left[2 \times \frac{1}{12} \times t(4t)^3 + \frac{1}{12} \times 3t(t)^3\right] = \frac{131}{12}t^4$$

$$\frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

- Since the value of I_{xx}/ I_{yy} lies between 3 and 3.5, therefore I–section chosen is quite Satisfactory.

Notes:

1. The I – section of the connecting rod is used due to its lightness and to keep the inertia forces as low as possible. It can also withstand high gas pressure.
2. Sometimes a connecting rod may have rectangular section. For slow speed engines, circular sections may be used.
3. Since connecting rod is manufactured by forging, therefore the sharp corners of I – section are rounded off as shown in Fig. 7.5 (b) for easy removal of the section from the dies.

7.11 Buckling of Columns

- A column or strut is a slender machine component that has considerable length in proportion to its width, depth or diameter. Column is also called strut, pillar or stanchion. Piston rod in hydraulic or pneumatic cylinder, push rod of valve mechanism, power screw in jack and connecting rod are the examples of columns.
- When a short member is subjected to axial compressive force, as shown in *Fig.7.6*(a), it shortens according to Hooke's law. As the load is gradually increased, the compression of the member increases. When the compressive stress reaches the elastic limit of the material, the failure occurs in the form of bulging.
- However, when the length of the component is large compared with the cross – sectional dimensions, as shown in *Fig.7.6*(b), the component may fail by lateral buckling. Buckling indicates elastic instability. The load at which the buckling starts is called critical load, which is denoted by Pcr. When the axial load on the column reaches Pcr, there is sudden buckling and relatively large deflection occurs. Some of the rules of thumb for buckling of columns are as follows:

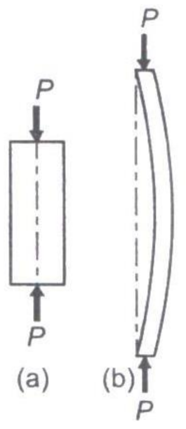


Fig.7.6

   i. A column made of ductile material, like steel and whose length is more than eight times of its least lateral dimension is likely to buckle and should be treated as column.
   ii. A column made of brittle material, like cast iron and whose length is more than six times of its least lateral dimension is likely to buckle and should be treated as column.

- There is a basic difference between the lateral deflection of a beam and the buckling of columns. The lateral deflection of the beam is gradually increased as the lateral load is increased. In case of buckling, there is no such lateral deflection till the load reaches the critical load. At this point, there is sudden lateral deflection that results in collapse of the column. The failure due to buckling is, therefore, sudden and total without any warning.
- An important parameter affecting the critical load is the slenderness ratio. It is defined as,

Eq. (7.2)

$$Slenderness\ Ration = \frac{l}{k}$$

Where,
l = length of the column (mm)
k = least radius of gyration of the cross – section about its axis (mm)

The radius of gyration is given by,

Eq. (7.3)

$$k = \sqrt{\frac{I}{A}}$$

Where,
I = least moment of inertia of the cross – section (mm^4)
A = area of the cross – section (mm^2)

- When the slenderness ratio is less than 30, there is no effect of buckling and such components are designed on the basis of compressive stresses. Columns, with slenderness ratio greater than 30, are designed on the basis of critical load.
- There are two terms namely, short and long columns, that are frequently used in buckling analysis. The rules of thumb for deciding long and short columns are as follows:

I. Cast iron columns with a slenderness ratio not greater than 80 and steel columns with a slenderness ratio not greater than 100, are considered short column.

II. Long columns are those with slenderness ratio greater than 100 for ductile materials and greater than 80 for cast iron.

There are two methods to calculate the critical load – Euler's equation and Johnson's equation.

According to Euler's equation,

$$P_{cr} = \frac{n\pi^2 EA}{\left({}^{l}/_{k}\right)^2} \qquad \textit{Eq. (7.4)}$$

Where,
P_{cr} = critical load (N)
n = end fixity coefficient
E = modulus of elasticity (N/mm^2)
A = area of the cross – section (mm^2)

- The load carrying capacity of the column depends upon the condition of restraints at the two ends of the column. It is accounted by means of a dimensionless quantity called end fixity coefficient (n). The values of n are listed in ***Table** 7.4*.

Table 7.4

Sr. No.	End condition	n
1	Both ends hinged	1
2	Both ends fixed	4
3	One end fixed and other end hinged	2
4	One end fixed and other end free	0.25

According to Johnson's equation,

$$P_{cr} = S_{yt}\, A\left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right] \qquad \textit{Eq. (7.5)}$$

- Where, S_{yt} is the yield strength of the material.
- In order to study the above two equations, we will consider a numerical example. Let us consider a column with both ends hinged and made of steel 45C8 (S_{yt} = 380 N/mm^2 and E = 207000 N/mm^2). According to Euler's equation,

$$\left(\frac{P_{cr}}{A}\right)_1 = \frac{n\pi^2 E}{\left({}^{l}/_{k}\right)^2} = \frac{(1)\pi^2(207000)}{\left({}^{l}/_{k}\right)^2} = \frac{2043008}{\left({}^{l}/_{k}\right)^2} \qquad \textit{Eq. (7.6)}$$

According to Johnson's equation,

$$\left(\frac{P_{cr}}{A}\right)_1 = S_{yt}\left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right] \qquad \textit{Eq. (7.7)}$$

$$= 380\left[1 - \frac{380}{4(1)\pi^2(207000)}\left(\frac{l}{k}\right)^2\right]$$

$$= 380\left[1 - \frac{\left({}^{l}/_{k}\right)^2}{21505}\right]$$

- The values of (P_{cr} / A) for different values of slenderness ratio (l/k) are tabulated as in ***Table 7.5***.
- The graph of unit load (P_{cr} / A) against slenderness ratio (l/k) is shown in *Fig.7.7*. The following observations are made from the figure.
  I. When the slenderness ratio is 60, the unit load according to Euler's equation is 568 N/mm^2, while the yield strength of the material is 380 N/mm^2. Therefore, Euler's equation is illogical in this range.
  II. When the slenderness ratio is more than 150, Johnson's equation gives negative values of unit load, which is illogical.
  III. The curves given by Euler's and Johnson's equations are tangential at point P, where unit load (P_{cr}/A) is equal to (S_{yt}/2), i.e. 190 N/mm^2. The slenderness ratio at this point can be considered as the boundary line between short and long columns.

Table 7.5

(l / k)	**60**	**80**	**100**	**103.7**	**120**	**140**	**160**	**180**
$(P_{cr}/A)_1$	568	319	204	190	142	104	80	63
$(P_{cr}/A)_2$	316	267	203	190	126	34	-	-

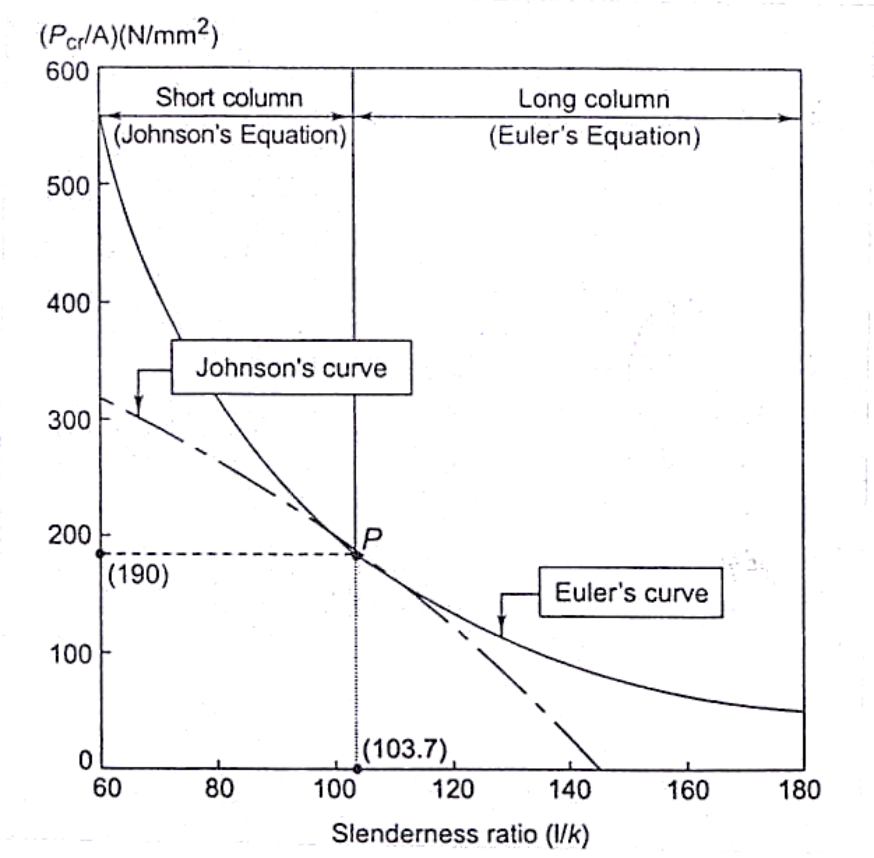


Fig.7.7 – Variation of unit load against slenderness ratio – Euler's & Johnson's criteria

- In design analysis, the question always arises as to whether one should use Euler's equation or Johnson's equation. From the above observations, it is concluded that Euler's equation is suitable for long columns, while Johnson's equation for short columns. The boundary line between the two is defined by equating unit load (P_{cr} / A) to half the yield strength.

$$\frac{P_{cr}}{A} = \frac{S_{yt}}{2} \qquad \textit{Eq. (7.8)}$$

From Eq. (7.4),

$$\frac{P_{cr}}{A} = \frac{n\pi^2 E}{\left(l/_k\right)^2} \qquad \textit{Eq. (7.9)}$$

From Eq. (7.8) and Eq. (7.9),

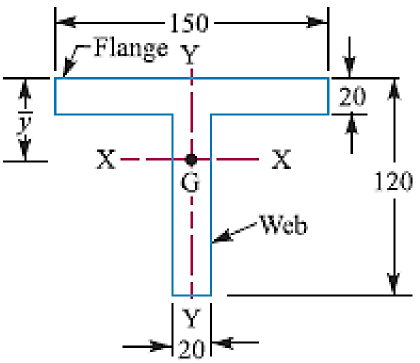
$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{\left(l/_k\right)^2} \qquad \textit{Eq. (7.10)}$$

The ratio (l /k) obtained by equation (v) is critical slenderness ratio between long and short columns. When actual slenderness ratio is less than the critical slenderness ratio, Johnson's equation is used. When the actual slenderness ratio is more than the critical slenderness ratio, Euler's equation should be used.

Ex. 7.1 ***[R.S. Khurmi; Example No. 16.1]***

A T – section 150 mm X 120 mm X 20 mm is used as a strut of 4 m long hinged at both ends. Calculate the crippling load, if Young's modulus for the material of the section is 200 kN/mm2.

Solution:



<u>Given Data:</u>

$l = 4\ m$

$E = 200\ kN/mm^2$

<u>To be Calculated:</u>

a) $W_{cr} = ?$

First of all, let us find the centre of gravity (G) of the T-section as shown in above Figure.

Let ȳ be the distance between the centre of gravity (G) and top of the flange,

We know that area of flange,

$$a_1 = 150 \times 20 = 3000\ mm^2$$

Its distance of centre of gravity from top of the flange,

$$y_1 = 20/2 = 10\ mm$$

Area of web,

$$a_2 = (120 - 20) \times 20 = 2000\ mm^2$$

Its distance of centre of gravity from top of the flange,

$$y_2 = 20 + (100/2) = 70\ mm$$

$$\bar{y} = \frac{a_1\, y_1 + a_2\, y_2}{a_1 + a_2} = \frac{3000 \times 10 + 2000 \times 70}{3000 + 2000}$$

$$= 34\ mm$$

We know that the moment of inertia of the section about X –X,

$$I_{xx} = \left[\frac{150(20)^3}{12} + 3000(34-10)^2 + \frac{20(100)^3}{12} + 2000(70-34)^2\right]$$

$$= 6.1 \times 10^6\ \ mm^4$$

$$I_{yy} = \frac{20(150)^3}{12} + \frac{100(20)^3}{12} = 5.7 \times 10^6 \ mm^4$$

Since I_{yy} is less than I_{xx} , therefore the column will tend to buckle in Y – Y direction. Thus we shall take the value of I as $I_{yy} = 5.7 \times 10^6\ mm^4$.

Moreover as the column is hinged at its both ends, therefore equivalent length,

$$L = l = 4000\ mm$$

We know that the crippling load,

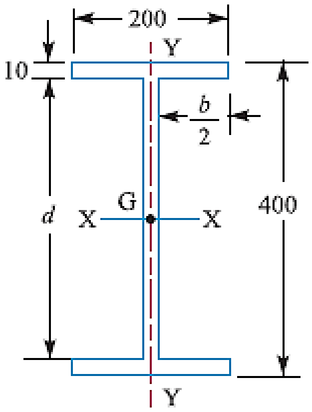
$$W_{cr} = \frac{\pi^2 E\ I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times 5.7 \times 10^6}{(4000)^2} = 703 \times 10^3 \ N$$

$$= 703\ kN$$

Ex. 7.2 ***[R.S. Khurmi; Example No. 16.2]***

An I – section 400 mm x 200 mm x 10 mm and 6 m long is used as a strut with both ends fixed. Find Euler's crippling load. Take Young's modulus for the material of the section as 200 kN/mm².

Solution:



Given Data:

$D = 400\ mm$

$B = 200\ mm$

$t = 10\ mm$

$l = 6\ m$

$E = 200\ kN/mm^2$

To be Calculated:

a) $W_{cr} = ?$

We know that the moment of inertia of the I – section about X – X,

$$I_{xx} = \left[\frac{B(D)^3}{12} - \frac{b(d)^3}{12}\right]$$

$$I_{xx} = \left[\frac{200(400)^3}{12} - \frac{(200-10)(400-20)^3}{12}\right]$$

$$= 200 \times 10^6 \ mm^4$$

and moment of inertia of the I – section about Y – Y,

$$I_{yy} = 2\left(\frac{tB^3}{12}\right) + \frac{dt^3}{12}$$

$$= 2\left(\frac{10(200)^3}{12}\right) + \frac{(400-20)10^3}{12}$$

$$= 13.36 \times 10^6 \ mm^4$$

Since I_{yy} is less than I_{xx} , therefore the section will tend to buckle about Y-Y axis. Thus we shall take I as $I_{yy} = 13.36 \times 10^6\ mm^4$.

Since the column is fixed at its both ends, therefore equivalent length,

$$L = \frac{l}{2} = \frac{6000}{2} = 3000\ mm$$

We know that the crippling load,

$$W_{cr} = \frac{\pi^2 E\ I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times 13.36 \times 10^6}{(3000)^2} = 2.93 \times 10^6\ \ N$$

$$= 2930\ kN$$

Ex. 7.3 ***[R.S. Khurmi; Example No. 16.3]***

Calculate the diameter of a piston rod for a cylinder of 1.5 m diameter in which the greatest difference of steam pressure on the two sides of the piston may be assumed to be 0.2 N/mm^2. The rod is made of mild steel and is secured to the piston by a tapered rod and nut and to the crosshead by a cotter. Assume modulus of elasticity as 200 kN/mm^2 and factor of safety as 8. The length of rod may be assumed as 3 metres.

Solution:

Given Data:

$D = 1.5\ m$

$p = 0.2\ MPa$

$E = 200\ kN/mm^2$

$l = 3\ m$

To be Calculated:

a) $d = ?$

We know that the load acting on the piston,

$$W = \frac{\pi}{4} \times D^2 \times p$$

$$= \frac{\pi}{4} \times (1500)^2 \times 0.2$$

$$= 353475\ N$$

Buckling load on the piston rod,

$$W_{cr} = w \times Factor\ of\ safety$$

$$= 353475 \times 8$$

$$= 2.83 \times 10^6\ \ N$$

Since the piston rod is considered to have both ends fixed, therefore from ***Table 7.2***, the equivalent length of the piston rod,

$$L = \frac{l}{2} = \frac{3000}{2} = 1500\ mm$$

Let d = Diameter of piston rod in mm, and

I = Moment of inertia of the cross – section of the rod

$$= \frac{\pi}{64} \times d^4$$

According to Euler's formula, buckling load (W_{cr}),

$$2.83 \times 10^6 = \frac{\pi^2 E\ I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times \pi d^4}{(1500)^2 \times 64} = \ 0.043 d^4$$

$$d^4 = 2.83 \times 10^6 / 0.043 = 65.8 \times 10^6$$

$$\boldsymbol{d = 90\ mm}$$

According to Rankine's formula, buckling load,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k}\right)^2} \qquad \textit{Eq. (7.11)}$$

We know that for mild steel, the crushing stress,

$$\sigma_c = 320\ MPa = 320\ N/mm^2 \quad and \ \ a = \frac{1}{7500}$$

and least radius of gyration for the piston rod section,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi d^4}{64} \times \frac{4}{\pi d^2}} = \frac{d}{4}$$

Substituting these values in the above *Eq. (7.11)*, we have

$$2.83 \times 10^6 = \frac{320 \times \frac{\pi d^4}{4}}{1 + \frac{1}{7500}\left(\frac{1500 \times 4}{d}\right)^2} = \frac{251.4\ d^2}{1 + \frac{4800}{d^2}} = \frac{251.4\ d^4}{d^2 + 4800}$$

$$251.4\ d^4 - 2.83 \times 10^6\ d^2 - 2.83 \times 10^6 \times 4800 = 0$$

$$d^4 - 11257\ d^2 - 54 \times 10^6 = 0$$

$$d^2 = \frac{11250 \pm \sqrt{(11257)^2 + (4 \times 1 \times 54 \times 10^6)}}{2}$$

$$d = 122\ mm$$

Taking larger of the two values, we have

$$\boldsymbol{d = 122\ mm}$$

Ex. 7.4 ***[R.S. Khurmi; Example No. 16.4]***

The maximum load on a petrol engine push rod 300 mm long is 1400 N. It is hollow having the outer diameter 1.25 times the inner diameter. Spherical seated bearings are used for the push rod. The modulus of elasticity for the material of the push rod is 210 kN / mm^2. Find a suitable size for the push rod, taking factor of safety of 2.5.

Solution:

Given Data:

$l = 300\ mm$

$W = 1400\ N$

$D = 1.25\ d$

$E = 200\ kN/mm^2$

To be Calculated:

a) $d = ?$

b) $D = ?$

Let d = Inner diameter of push rod in mm, and
D = Outer diameter of the push rod in mm = 1.25 d ….(Given)

Moment of inertia of the push rod section,

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}[(1.25d)^4 - d^4)]$$

$$= 0.07\ d^4\ \ mm^4$$

We know that the crippling load on the push rod,

$$W_{cr} = m \times W = 2.5 \times 1400 = 3500\ N$$

Now according to Euler's formula, crippling load (W_{cr}),

$$3500 = \frac{\pi^2 E\ I}{L^2} = \frac{9.87 \times 210 \times 10^3 \times 0.07\ d^4}{(300)^2} = 1.6\ d^4$$

$$d^4 = \frac{3500}{1.6} = 2188$$

$$\boldsymbol{d = 6.84\ mm}$$

and

$$D = 1.25\ d = 1.25 \times 6.84$$

$$\boldsymbol{D = 8.55\ mm}$$

Ex. 7.5 ***[R.S. Khurmi; Example No. 16.5]***

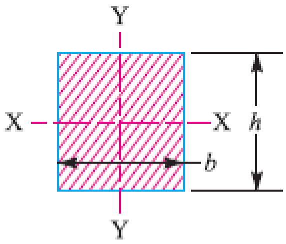
A connecting rod of length l may be considered as a strut with the ends free to turn on the crank pin and the gudgeon pin. In the directions of the axes of these pins, however, it may be considered as having fixed ends. Assuming that Euler's formula is applicable, determine the ratio of the sides of the rectangular cross – section so that the connecting rod is equally strong in both planes of buckling.

Solution: Let b = Width of rectangular cross – section, and

h = Depth of rectangular cross – section.

Moment of inertia about X – X

$$I_{xx} = \frac{bh^3}{12}$$



Moment of inertia about Y – Y

$$I_{yy} = \frac{hb^3}{12}$$

According to Euler's formula, buckling load,

$$W_{cr} = \frac{\pi^2 E\ I}{L^2}$$

∴ Buckling load about X – X,

$$W_{cr}\ (X - axis) = \frac{\pi^2 E\ I_{xx}}{l^2} \qquad (L = l, for\ both\ ends\ free\ to\ turn)$$

and buckling load about Y – Y,

$$W_{cr}\ (Y - axis) = \frac{\pi^2 E\ I_{yy}}{(l/2)^2} = \frac{4\pi^2 E\ I_{yy}}{(l)^2} \qquad (L = l/2, for\ both\ ends\ fixed)$$

In order to have the connecting rod equally strong in both the planes of buckling,

$$W_{cr}\ (X - axis) = W_{cr}\ (Y - axis)$$

$$\frac{\pi^2 E\ I_{xx}}{l^2} = \frac{4\pi^2 E\ I_{yy}}{(l)^2} \qquad or \qquad I_{xx} = 4I_{yy}$$

$$\frac{bh^3}{12} = \frac{4hb^3}{12} \qquad or \qquad h^2 = 4b^2$$

$$h^2/b^2 = 4 \qquad or \qquad h/b = 2$$

Ex. 7.6 ***[V.B. Bhandari; Example No. 23.5]***

A 25 x 50 mm bar of rectangular cross – section is made of plain carbon steel 40C8 (Syt = 380 N/mm^2 and E = 207 000 N/mm^2). The length of the bar is 500 mm. The two ends of the bar are hinged and the factor of safety is 2.5. The bar is subjected to axial compressive force.

i. Determine the slenderness ratio;
ii. Which of the two equations – Euler's or Johnson's will you apply to the bar?
iii. What is the safe compressive force for the bar?

Solution: ***Given Data:***

$S_{yt} = 380\ N/mm^2$

$E = 207000\ N/mm^2$

$l = 500\ mm$

$F.O.S. = 2.5$

$$I = \frac{(50)(25)^3}{12}\ mm^4$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{(50)(25)^3}{12(50 \times 25)}} = 7.22\ mm$$

$$\left(\frac{l}{k}\right) = \frac{500}{7.22} = 69.25$$

The boundary line between Johnson's and Euler's equation is given by,

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{\left({}^{l}/_{k}\right)^2}$$

$$\frac{380}{2} = \frac{(1)\pi^2(207000)}{\left({}^{l}/_{k}\right)^2}$$

$$\left(\frac{l}{k}\right) = 103.7$$

Since the slenderness ratio of the bar (69.25) is less than 103.7, the bar is treated as a short column and Johnson's equation is applicable.

$$P_{cr} = S_{yt}\ A\left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right]$$

$$P_{cr} = (380)(25 \times 50)\left[1 - \frac{380}{4(1)\pi^2(207000)}(69.25)^2\right]$$

$$P_{cr} = 369077.88\ N$$

The safe compressive force is given by,

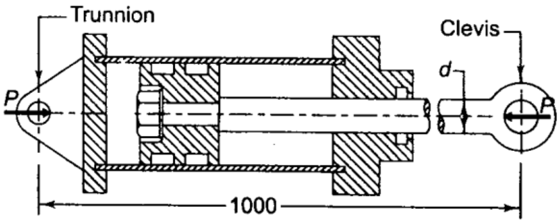
$$P = \frac{P_{cr}}{F.O.S.} = \frac{369077.88}{2.5}$$

$$P = 147631.15 \ N$$

Ex. 7.7 ***[V.B. Bhandari; Example No. 23.6]***

A trunnion mounted hydraulic cylinder is shown in below figure. The internal diameter of the cylinder is 75mm and the maximum operating pressure in the cylinder 25 N/mm^2. The piston rod is made of steel 40Cr1 (S_{yt} = 530 N/mm^2 and E = 207 000 N/mm^2). For buckling considerations, the effective length of the piston rod is considered as the distance between the trunnion and the clevis – mount, when the piston rod is extended to its full working stroke and this distance is 1000 mm. Determine the diameter of the piston rod, if the factor of safety is 2.5.

Solution:



Given Data:

$D = 75 \ mm$

$p = 25 \ N/mm^2$

$S_{yt} = 530 \ N/mm^2$

$E = 207000 \ N/mm^2$

To be Calculated:

a) $d = ?$

Although one end of the piston rod is fixed in the piston, considering the complete assembly between trunion and clevis – mount, the end fixity coefficient is taken as one (both ends hinged). The maximum force on the piston rod is given by,

$$P = \frac{\pi}{4} D^2 \, p = \frac{\pi}{4}(75)^2(25) = 110446.6 \ N$$

Using a factor of safety of 2.5,

$$P_{cr} = 2.5 \, P = 2.5(110446.6) = 276116.5 \, N$$

For circular cross – section,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{(\pi d^4)/64}{(\pi d^2)/4}} = \left(\frac{d}{4}\right) mm$$

At this stage, it is not clear whether, one should use Euler's or Johnson's equation. Using euler's equation as a first trial,

$$P_{cr} = \frac{n\pi^2 EA}{\left({}^{l}/_{k}\right)^2}$$

$$276116.5 = \frac{(1)\pi^2(207000)(\pi d^2/4)}{\left(\frac{1000}{d/4}\right)^2}$$

$$\boldsymbol{d = 40.73 \ mm}$$

$$\left(\frac{l}{k}\right) = \frac{1000}{(40.73/4)} = 98.21$$

The boundary line between Euler's and Johnson's equation is given by

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{\left(l/_{k}\right)^2}$$

$$\frac{530}{2} = \frac{(1)\pi^2(207000)}{\left(l/_{k}\right)^2}$$

$$\left(\frac{l}{k}\right) = 87.8$$

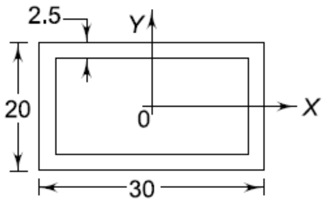
In this example, the slenderness ratio (98.21) is greater than the boundary value of (87.8). Therefore the assembly is treated as a long column and Euler's equation used in the first trial is justified.

$$\boldsymbol{d = 40.73\ mm}$$

Ex. 7.8 ***[V.B. Bhandari; Example No. 23.7]***

A column of hollow rectangular cross – section and made of steel plates is shown in below figure. The thickness of the section is 2.5 mm throughout. The plate material is steel 30C8 (S_{yt} = 400 N/mm^2 and E = 207 000 N/mm^2). The end fixity coefficients can be taken as 1.5 and 1 for bending about long and short axes respectively. The effective length of the column is 1 m. Determine the load capacity of the column from buckling consideration.

Solution:



Given Data:

$S_{yt} = 400\ N/mm^2$

$E = 207000\ N/mm^2$

$l = 1\ m$

To be Calculated:

a) $P_{cr} = ?$

$$A = (30 \times 20) - (25 \times 15) = 225\ mm^2$$

$$I_{xx} = \frac{1}{12}[30(20)^3 - 25(15)^3] = 12968.75\ mm^4$$

$$I_{yy} = \frac{1}{12}[20(30)^3 - 15(25)^3] = 25468.75\ mm^4$$

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{12968.75}{225}} = 7.59\ mm$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{25468.75}{225}} = 10.64\ mm$$

Eq. (7.12)

$$\left(\frac{l}{k_{xx}}\right) = \frac{1000}{7.59} = 131.75$$

Eq. (7.13)

$$\left(\frac{l}{k_{yy}}\right) = \frac{1000}{10.64} = 93.98$$

For the XX – axis,

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{\left(^{l}/_{k}\right)^2}$$

$$\frac{400}{2} = \frac{(1.5)\pi^2(207000)}{\left(^{l}/_{k_{xx}}\right)^2}$$

$$\left(\frac{l}{k_{xx}}\right) = 123.78$$ *Eq. (7.14)*

From Eq. (7.12) and Eq. (7.14), the column is treated as a long column. Using Euler's equation,

$$P_{cr} = \frac{n\pi^2 EA}{\left(^{l}/_{k_{xx}}\right)^2} = \frac{(1.5)\pi^2(207000)(225)}{(131.75)^2}$$

$$P_{cr} = 39723.05\ N$$ *Eq. (7.15)*

For the YY – axis,

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{\left(^{l}/_{k}\right)^2}$$

$$\frac{400}{2} = \frac{(1)\pi^2(207000)}{\left(^{l}/_{k_{yy}}\right)^2}$$

$$\left(\frac{l}{k_{yy}}\right) = 101.07$$ *Eq. (7.16)*

From *Eq. (7.13)* and *Eq. (7.16)*, the column is treated as a short column. Using Johnson's equation,

$$P_{cr} = S_{yt}\,A\left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k_{yy}}\right)^2\right]$$

$$P_{cr} = (400)(225)\left[1 - \frac{400}{4(1)\pi^2(207000)}(93.98)^2\right]$$

$$P_{cr} = 51091.61\ N$$ *Eq. (7.17)*

From *Eq. (7.15)* and *Eq. (7.17)*, the load carrying of the column is **39723.05 N**.

Ex. 7.9 ***[V.B. Bhandari; Example No. 23.8]***

It is required to design the screw of a screw – jack by buckling consideration. One end of the screw is fixed in the nut and the other end supports a load of 20 kN. The length of the screw between the fixed and free ends is 500 mm, when the load is completely raised. The screw is made of steel 40C8 (S_{yt} = 380 N/mm^2 and E = 207 000 N/mm^2). Assuming a factor of safety 2.5, determine the core diameter of the screw.

Solution: ***Given Data:***

$P_{cr} = 20\ kN$

$l = 500\ mm$

$S_{yt} = 380\ N/mm^2$

$E = 207000\ N/mm^2$

$F.O.S = 2.5$

To be Calculated:

a) $d = ?$

The end fixity coefficient is 0.25 when one end is fixed and the other free. As a first trial, using Euler's equation,

$$P_{cr} = \frac{n\pi^2 EA}{\left(l/k\right)^2}$$

$$(20000)(2.5) = \frac{(0.25)\pi^2(207000)((\pi d^2)/4)}{\left[\frac{500}{(d/4)}\right]^2}$$

$$d = 26.57\ mm$$

$$k = d/4 = 6.64\ mm$$

$$\left(\frac{l}{k}\right) = \frac{500}{6.64} = 75.3$$ *Eq. (7.18)*

The border line between Euler's and Johnson's equations is given by

$$\frac{S_{yt}}{2} = \frac{n\pi^2 E}{\left(l/k\right)^2}$$

$$\frac{380}{2} = \frac{(0.25)\pi^2(207000)}{\left(l/k\right)^2}$$

$$\left(\frac{l}{k}\right) = 51.85$$ *Eq. (7.19)*

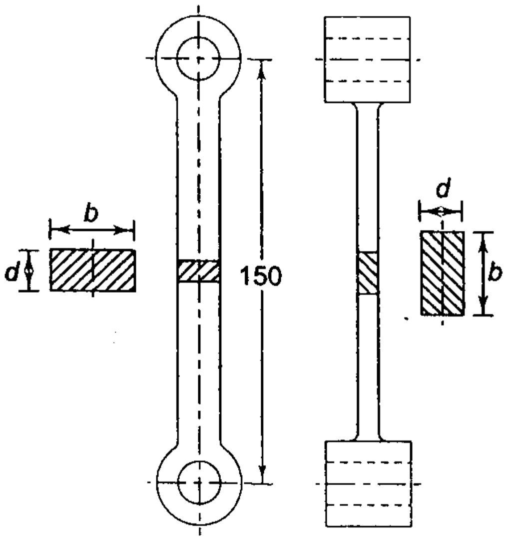
From Eq. (7.18) and Eq. (7.19), it is concluded that the screw is a long column and Euler's equation considered in the first trial is justified. Therefore,

$$\boldsymbol{d = 26.57\ mm}$$

Ex. 7.10 ***[V.B. Bhandari; Example No. 23.9]***

A piston rod of rectangular cross – section, with both ends hinged, is shown in figure. It is made of steel 40C8 (S_{yt} = 380 N/mm^2 and E = 207 000 N/mm^2) and subjected to an axial compressive force of 15 kN. Determine the ratio of (b/d) for equal buckling strength in either plane. Also determine the dimensions of cross – section, if the factor of safety is 4.

Solution:



Plane-A Plane-B

Given Data:

$S_{yt} = 380\ N/mm^2$

$E = 207000\ N/mm^2$

$P_{cr} = 15\ kN$

$F.O.S. = 4$

To be Calculated:

a) $b/d = ?$

b) $b = ?$

For the purpose of convenience, the planes are called A and B as shown in above figure. In plane – A, the ends are hinged.

$$n_A = 1 \qquad and \qquad I_A = \left(\frac{db^3}{12}\right)$$

In plane – B, the ends are fixed.

$$n_B = 4 \qquad and \qquad I_B = \left(\frac{bd^3}{12}\right)$$

Using Euler's equation and equating buckling load in two planes,

$$\frac{n_A \pi^2 EA}{\left(\frac{l}{k_A}\right)^2} = \frac{n_B \pi^2 EA}{\left(\frac{l}{k_B}\right)^2}$$

$$n_A (k_A)^2 = n_B (k_B)^2$$

$$Substituting \qquad k^2 = I/A$$

$$n_A\, I_A = n_B\, I_B$$

$$(1)\left(\frac{db^3}{12}\right) = (4)\left(\frac{bd^3}{12}\right)$$

$$\left(\frac{\boldsymbol{b}}{\boldsymbol{d}}\right) = \mathbf{2}$$

As the first trial, using Johnson's equation in plane A,

$$(k_A)^2 = \frac{I_A}{A} = \left(\frac{bd^3}{12}\right)\left(\frac{1}{bd}\right) = \left(\frac{b^2}{12}\right)$$

$$\left(\frac{l}{k_A}\right)^2 = \frac{(150)^2}{\left(\frac{b^2}{12}\right)} = \frac{270000}{b^2}$$

$$P_{cr} = S_{yt}\, A\left[1 - \frac{S_{yt}}{4n\pi^2 E}\left(\frac{l}{k}\right)^2\right]$$

$$(15 \times 10^3 \times 4) = (380)(0.5b^2)\left[1 - \frac{380}{4(1)\pi^2(207000)}\left(\frac{270000}{b^2}\right)\right]$$

$$\boldsymbol{b = 18.12\ mm}$$

$$\left(\frac{l}{k_A}\right)^2 = \frac{270000}{b^2} = \frac{270000}{(18.12)^2} = 26.68$$

Since the slenderness ratio is small, Johnson's equation is justified.

7.12 References

1) A Textbook of Machine Design by R.S. Khurmi, S. Chand Publication.
2) Strength of Materials by R.S. Khurmi, S. Chand Publication.
3) Design of Machine Elements by V.B. Bhandari, McGraw-Hill Publication.