8

Shafts and Keys

Contents

8.1 Introduction to Shaft

- A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.
- In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.
- In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.
- An axle, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.
- A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

8.1.1 Materials and Manufacturing of Shafts

- The material used for shafts should have the following properties:
    1. It should have high strength.
    2. It should have good machinability.
    3. It should have low notch sensitivity factor.
    4. It should have good heat treatment properties.
    5. It should have high wear resistant properties.
- The material used for ordinary shafts is carbon steel of grades 40C8, 45C8, 50C4 and 50C12.
- Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding.
- The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut.
- Shafts of larger diameter are usually forged and turned to size in a lathe.

8.1.2 Types of Shafts

- The following two types of shafts are important from the subject point of view:
    1. **Transmission shafts:** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.
    2. **Machine shafts:** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.
- The standard lengths of the shafts are 5 m, 6 m and 7 m.

8.2 Design of Shafts

- The shafts may be designed on the basis of
  1) Strength, 2) Rigidity and 3) Stiffness

8.2.1 Designing of Shafts on the Strength Basis

- In designing shafts on the basis of strength, the following cases may be considered :
  - (a). Shafts subjected to twisting moment or torque only,
  - (b). Shafts subjected to bending moment only,
  - (c). Shafts subjected to combined twisting and bending moments.

8.2.1.1 Shafts Subjected to Twisting Moment Only

- When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation.
- We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \textit{Eq. (8.1)}$$

where, T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

= d / 2; where d is the diameter of the shaft.

- We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} d^4$$

- The *Eq. (8.1)* may now be written as

$$\frac{T}{\frac{\pi}{32} d^4} = \frac{\tau}{\frac{d}{2}}$$

$$T = \frac{\pi}{16} \times \tau \times d^3 \qquad \textit{Eq. (8.2)}$$

- From this equation, we may determine the diameter of round solid shaft (d).
- We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[{d_O}^4 - {d_i}^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and r = d_o / 2.

- Substituting these values in *Eq. (8.1)*, we have

$$\frac{T}{\frac{\pi}{32}\left[{d_O}^4 - {d_i}^4\right]} = \frac{\tau}{\frac{d_O}{2}}$$

$$T = \frac{\pi}{16} \times \tau \left[\frac{{d_O}^4 - {d_i}^4}{d_O}\right] \qquad \textit{Eq. (8.3)}$$

- Let k = Ratio of inside diameter and outside diameter of the shaft = d_i / d_o
- Now the *Eq. (8.3)* may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{{d_O}^4}{d_O}\left[1 - \left(\frac{d_i}{d_O}\right)^4\right]$$

$$T = \frac{\pi}{16} \times \tau \times {d_O}^3\left[1 - (k)^4\right] \qquad \textit{Eq. (8.4)}$$

- From the *Eq. (8.3)* or *Eq. (8.4)*, the outside and inside diameter of a hollow shaft may be determined.

- It may be noted that
  1. The twisting moment (T) may be obtained by using the following relation:

     We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi NT}{60000}$$

$$T = \frac{60000P}{2\pi N}$$

Where T = Twisting moment in N-mm, and

N = Speed of the shaft in r.p.m.

  2. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\,R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

8.2.1.2 Shafts Subjected to Bending Moment Only

- When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation.
- We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \textit{Eq. (8.5)}$$

where, M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre = d / 2.

- We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} d^4$$

- The *Eq. (8.5)* may now be written as

$$\frac{M}{\frac{\pi}{64} d^4} = \frac{\sigma_b}{\frac{d}{2}}$$

$$M = \frac{\pi}{64} \times \sigma_b \times d^3 \qquad \textit{Eq. (8.6)}$$

- From this equation, we may determine the diameter of round solid shaft (d).
- We also know that for hollow shaft, polar moment of inertia,

$$I = \frac{\pi}{64}\left[{d_O}^4 - {d_i}^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and r = d_o / 2.

- Substituting these values in *Eq. (8.5)*, we have

$$\frac{M}{\frac{\pi}{64}\left[{d_O}^4 - {d_i}^4\right]} = \frac{\sigma_b}{\frac{d_O}{2}}$$

$$M = \frac{\pi}{64} \times \sigma_b \times \left[\frac{{d_O}^4 - {d_i}^4}{d_O}\right] \qquad \textit{Eq. (8.7)}$$

- Let k = Ratio of inside diameter and outside diameter of the shaft = d_i / d_o

- Now the *Eq. (8.7)* may be written as

$$M = \frac{\pi}{32} \times \sigma_b \times \frac{{d_O}^4}{d_O}\left[1 - \left(\frac{d_i}{d_O}\right)^4\right]$$

$$M = \frac{\pi}{32} \times \sigma_b \times {d_O}^3[1 - (k)^4] \qquad \textit{Eq. (8.8)}$$

- From the *Eq. (8.7)* or *Eq. (8.8)*, the outside and inside diameter of a hollow shaft may be determined.
- It may be noted that the axles are used to transmit bending moment only. Thus, axles are designed on the basis of bending moment only.

8.2.1.3 Shafts Subjected to Combined Twisting Moment and Bending Moment

- When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously.
- Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses.
- The following two theories are important from the subject point of view:
  1. Maximum shear stress theory or Guest's theory: It is used for ductile materials such as mild steel.
  2. Maximum normal stress theory or Rankine's theory: It is used for brittle materials such as cast iron.

  Let τ = Shear stress induced due to twisting moment, and
  σ_b = Bending stress (tensile or compressive) induced due to bending moment.

1. **Maximum shear stress theory or Guest's theory**: According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + (\tau)^2}$$

   – We know that

$$T = \frac{\pi}{16} \times \tau \times d^3 \qquad \therefore \tau = \frac{16T}{\pi d^3}$$

$$M = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad \therefore \sigma_b = \frac{32M}{\pi d^3}$$

   – Substituting the values of σ_b and τ in the above equation, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{(M)^2 + (T)^2}$$

$$\frac{\pi d^3}{16} \times \tau_{max} = \sqrt{(M)^2 + (T)^2} \qquad \textit{Eq. (8.9)}$$

   – The expression $M^2 + T^2$ is known as equivalent twisting moment and is denoted by Te.

   – The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment.

   – By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the *Eq. (8.9)* may be written as

$$T_e = \frac{\pi d^3}{16} \times \tau_{max} = \sqrt{(M)^2 + (T)^2}$$ *Eq. (8.10)*

– From this expression, diameter of the shaft (d) may be evaluated.

2. **Maximum normal stress theory or Rankine's theory**: According to maximum normal stress theory, the maximum normal stress in the shaft,

$$(\sigma_b)_{max} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + (\tau)^2}$$

$$(\sigma_b)_{max} = \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$(\sigma_b)_{max} = \frac{32}{\pi d^3} + \left[\frac{1}{2}\left(M + \sqrt{(M)^2 + (T)^2}\right)\right]$$

$$\frac{\pi d^3}{32}(\sigma_b)_{max} = \left[\frac{1}{2}\left(M + \sqrt{(M)^2 + (T)^2}\right)\right]$$ *Eq. (8.11)*

– The expression $\left[\frac{1}{2}\left(M + \sqrt{(M)^2 + (T)^2}\right)\right]$ is known as equivalent bending moment and is denoted by Me.

– The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.

– By limiting the maximum normal stress $(\sigma_b)_{max}$ equal to the allowable bending stress (σ_b), then the *Eq. (8.11)* may be written as

$$M_e = \frac{\pi}{32} \times \sigma_b \times d^3 = \left[\frac{1}{2}\left(M + \sqrt{(M)^2 + (T)^2}\right)\right]$$ *Eq. (8.12)*

From this expression, diameter of the shaft (d) may be evaluated.

– In case of a hollow shaft, the *Eq. (8.10)* and *Eq. (8.12)* may be written as

$$T_e = \frac{\pi}{16} \times \tau \times {d_O}^3[1 - (k)^4] = \sqrt{(M)^2 + (T)^2}$$

$$M_e = \frac{\pi}{32} \times \sigma_b \times {d_O}^3[1 - (k)^4] = \left[\frac{1}{2}\left(M + \sqrt{(M)^2 + (T)^2}\right)\right]$$

8.2.2 Designing of Shafts on the Rigidity Basis

- Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

8.2.2.1 Designing of shafts on the torsional rigidity basis

– The torsional rigidity is important in the case of camshaft of an I. C. engine where the timing of the valves would be affected.

– The permissible amount of twist should not exceed 0.25° per metre length of such shafts.

– For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value.

– The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

– The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \qquad or \qquad \frac{T}{J} = \frac{G.\theta}{L}$$

where θ = Torsional deflection or angle of twist in radians,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} d^4 \qquad (For\ solid\ shaft)$$

$$= \frac{\pi}{32} [(d_O)^4 - (d_i)^4] \qquad (For\ hollow\ shaft)$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

8.2.2.2 Designing of shafts on the lateral rigidity basis

- It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces.
- The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment.
- If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials.
- But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, i.e.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

8.3 Introduction to Keys

- A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.
- It is always inserted parallel to the axis of the shaft.
- Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.
- A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

8.4 Types of Keys

The following types of keys are important from the subject point of view :

1. Sunk keys,
2. Saddle keys,
3. Tangent keys,
4. Round keys, and
5. Splines.

8.4.1 Sunk keys,

- Sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types:

8.4.1.1 Rectangular sunk key

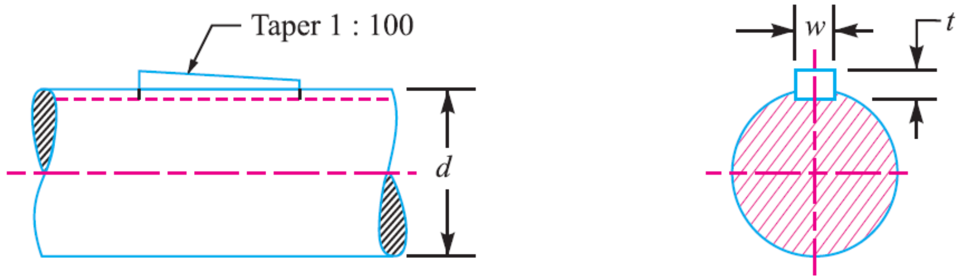


Fig.8.1 – Rectangular sunk key

– A rectangular sunk key is shown in *Fig.8.1*. The usual proportions of this key are :

Width of key, $w = {}^{d}/_{4}$; and thickness of key, $t = {}^{2w}/_{3} = {}^{d}/_{6}$

where d = Diameter of the shaft or diameter of the hole in the hub.

– The key has taper 1 in 100 on the top side only.

8.4.1.2 Square sunk key

– The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e. $w = t = {}^{d}/_{4}$.

8.4.1.3 Parallel sunk key

– The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout.

– It may be noted that a parallel key is a taper less and is used where the pulley, gear or other mating piece is required to slide along the shaft.

8.4.1.4 Gib-head key

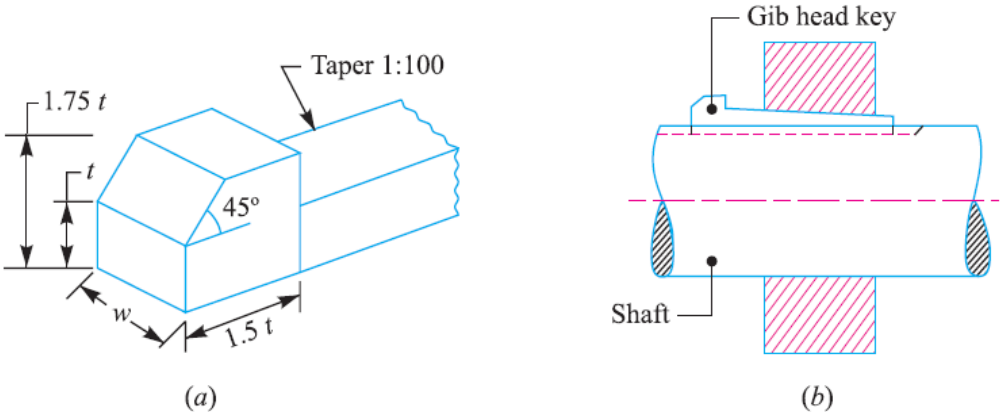


Fig.8.2 – Gib-head key

– It is a rectangular sunk key with a head at one end known as Gib head.

– It is usually provided to facilitate the removal of key. A Gib head key is shown in *Fig.8.9* (a) and its use in shown in *Fig.8.9* (b).

8.4.1.5 Feather key

– A key attached to one member of a pair and which permits relative axial movement is known as feather key.

– It is a special type of parallel key which transmits a turning moment and also permits axial movement.

– It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.
– The feather key may be screwed to the shaft as shown in *Fig.8.9* (a) or it may have double Gib heads as shown in *Fig.8.9* (b).
– The various proportions of a feather key are same as that of rectangular sunk key and Gib head key.

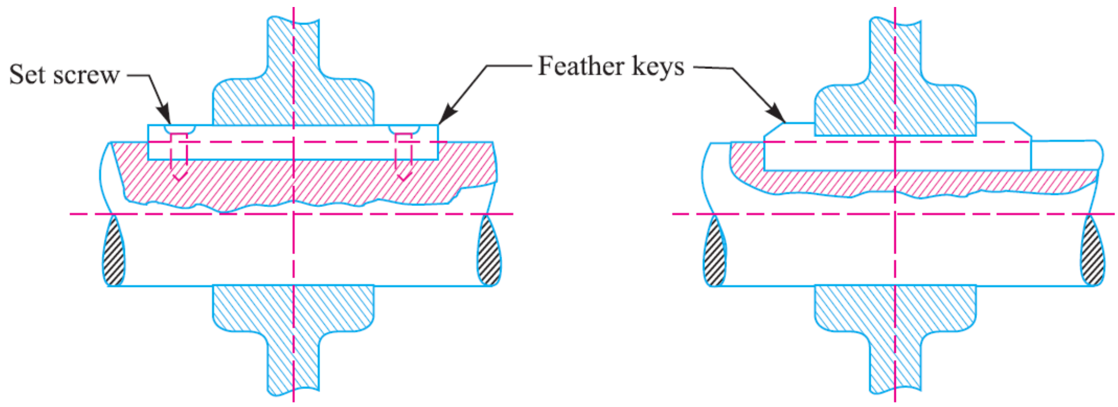


Fig.8.3 – Feather key

8.4.1.6 Woodruff key

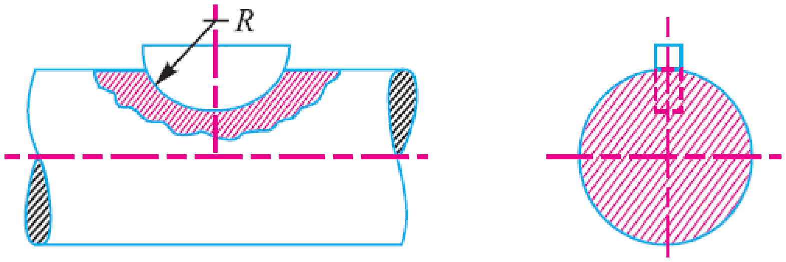


Fig.8.4 – Woodruff key

– The woodruff key is an easily adjustable key.

– It is a piece from a cylindrical disc having segmental cross-section in front view as shown in *Fig.8.9*.

– A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made.

– This key is largely used in machine tool and automobile construction.

– The main advantages of a woodruff key are as follows:
  (a). It accommodates itself to any taper in the hub or boss of the mating piece.
  (b). It is useful on tapering shaft ends. Its extra depth in the shaft prevents any tendency to turn over in its keyway.

– The disadvantages are :
  (a). The depth of the keyway weakens the shaft.
  (b). It cannot be used as a feather.

8.4.2 Saddle Keys

- The saddle keys are of the following two types: 1. Flat saddle key and 2. Hollow saddle key.
- A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in *Fig.8.9*.
- It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

- A hollow saddle key is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft.
- Since hollow saddle keys hold on by friction, therefore these are suitable for light loads.
- It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

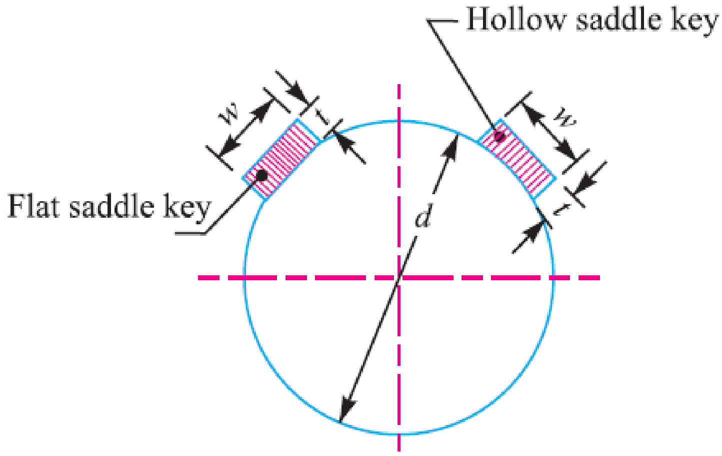


Fig.8.5 – Saddle key

8.4.3 Tangent Keys

- The tangent keys are fitted in pair at right angles as shown in *Fig.8.9*. Each key is to withstand torsion in one direction only.
- These are used in large heavy duty shafts.

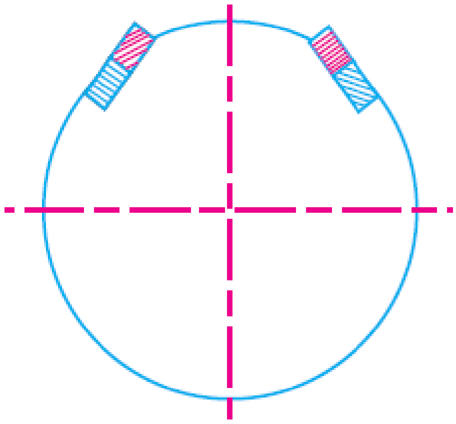


Fig.8.6 – Tangent key

8.4.4 Round Keys

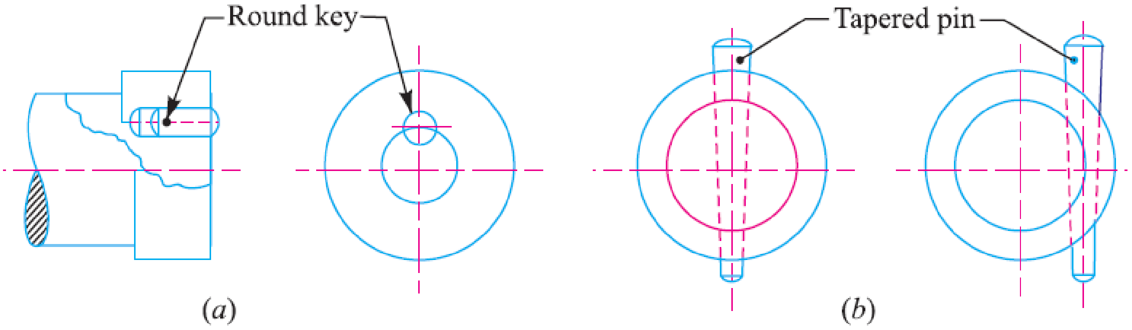


Fig.8.7 – Round keys

- The round keys, as shown in *Fig.8.9*(a), are circular in section and fit into holes drilled partly in the shaft and partly in the hub.
- They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled.
- Round keys are usually considered to be most appropriate for low power drives.
- Sometimes the tapered pin, as shown in *Fig.8.9* (b), is held in place by the friction between the pin and the reamed tapered holes.

8.4.5 Splines

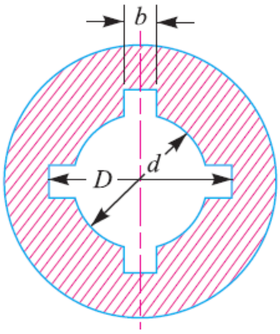


Fig.8.8 – Splines

- Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as splined shafts as shown in *Fig.8.9*.
- These shafts usually have four, six, ten or sixteen splines.
- The splined shafts are relatively stronger than shafts having a single keyway.
- The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions.
- By using splined shafts, we obtain axial movement as well as positive drive is obtained.

Ex. 8.1 Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.

If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter considering the ratio of inside to outside diameters as 0.5.

Solution:

Given Data:

P = 20 kW

N = 200 rpm

$\tau_u = 360\ MPa$

fs = 8

$k = \frac{d_i}{d_o} = 0.5$

To be Calculated:

d = ?

$d_i = ?$

$d_o = ?$

The allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45\ N/mm^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

Torque transmitted by the shaft,

$$T = \frac{60\,P}{2\pi N} = \frac{60\ \times 20000}{2\pi \times 200}$$

$$= 955\ N.m = 955 \times 10^3\ N.mm$$

Torque transmitted by the solid shaft (T),

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$955000 = \frac{\pi}{16} \times 45 \times d^3$$

$$\boldsymbol{d = 47.6\ \cong\ 50\ mm}$$

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

The torque transmitted by the hollow shaft (T),

$$T = \frac{\pi}{16} \times \tau \times {d_O}^3[1-(k)^4]$$

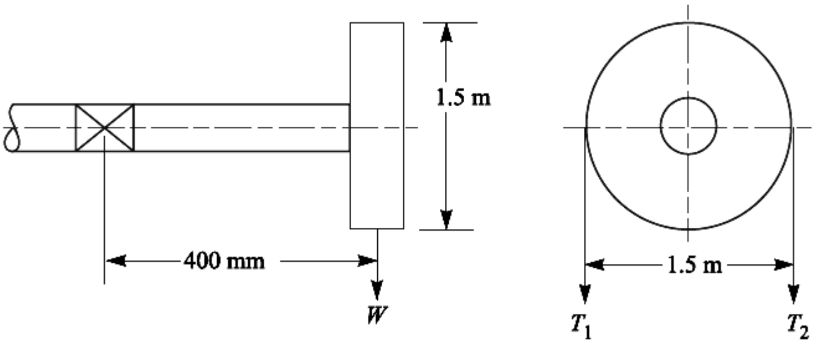
$$955000 = \frac{\pi}{16} \times 45 \times {d_O}^3[1-(0.5)^4]$$

$$\boldsymbol{d_O = 48.6\ \cong\ 50\ mm}$$

$$k = \frac{d_i}{d_O}$$

$$\therefore\ \boldsymbol{d_i = k.d_O = 0.5(50) = 25\ mm}$$

Ex. 8.2 A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assume maximum allowable shear stress of 42 MPa.



Solution: ***Given Data:***

D = 1.5 m

T_1 = 5.4 kN

T_2 = 1.8 kN

L = 400 mm

$\tau = 42\ MPa$

To be Calculated:

d = ?

The torque transmitted by the shaft,

$$T = (T_1 - T_2)R = (5400 - 1800)0.75$$

$$= 2700\ N.m = 2700 \times 10^3\ N.mm$$

Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200\ N$$

∴ Bending moment, M = W × L = 7200 × 400

$= 2880 \times 10^3$ N-mm

Let d = Diameter of the shaft in mm

We know that the equivalent twisting moment,

$$T_e = \sqrt{(M)^2 + (T)^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$

$$= 3950 \times 10^3\ N.mm$$

Equivalent twisting moment (T_e),

$$T_e = \frac{\pi}{16} \times \tau \times d^3$$

$$3950 \times 10^3 = \frac{\pi}{16} \times 42 \times d^3$$

$$\boldsymbol{d = 78 \cong 80\ mm}$$

Ex. 8.3 A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Solution:

Given Data:

AB = 1 m = 1000 mm;
AC = 300 mm = 0.3 m;
BD = 200 mm = 0.2 m;
T_1 = 2.25 kN =2250 N;
D_C = 600 mm or R_C = 300 mm
D_D = 400 mm or R_D = 200 mm

Given Data:

θ = 180° = π radian;
μ = 0.24;
σ_b = 63 MPa
τ = 42 MPa = 42 N/mm²

To be Calculated:

d =?

The space diagram of the shaft is shown in the *Fig.8.9* (a).

Let T_1 = Tension in the tight side of the belt on pulley C = 2250 N (given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that,

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{(0.24\times\pi)} = 2.127$$

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058\ N$$

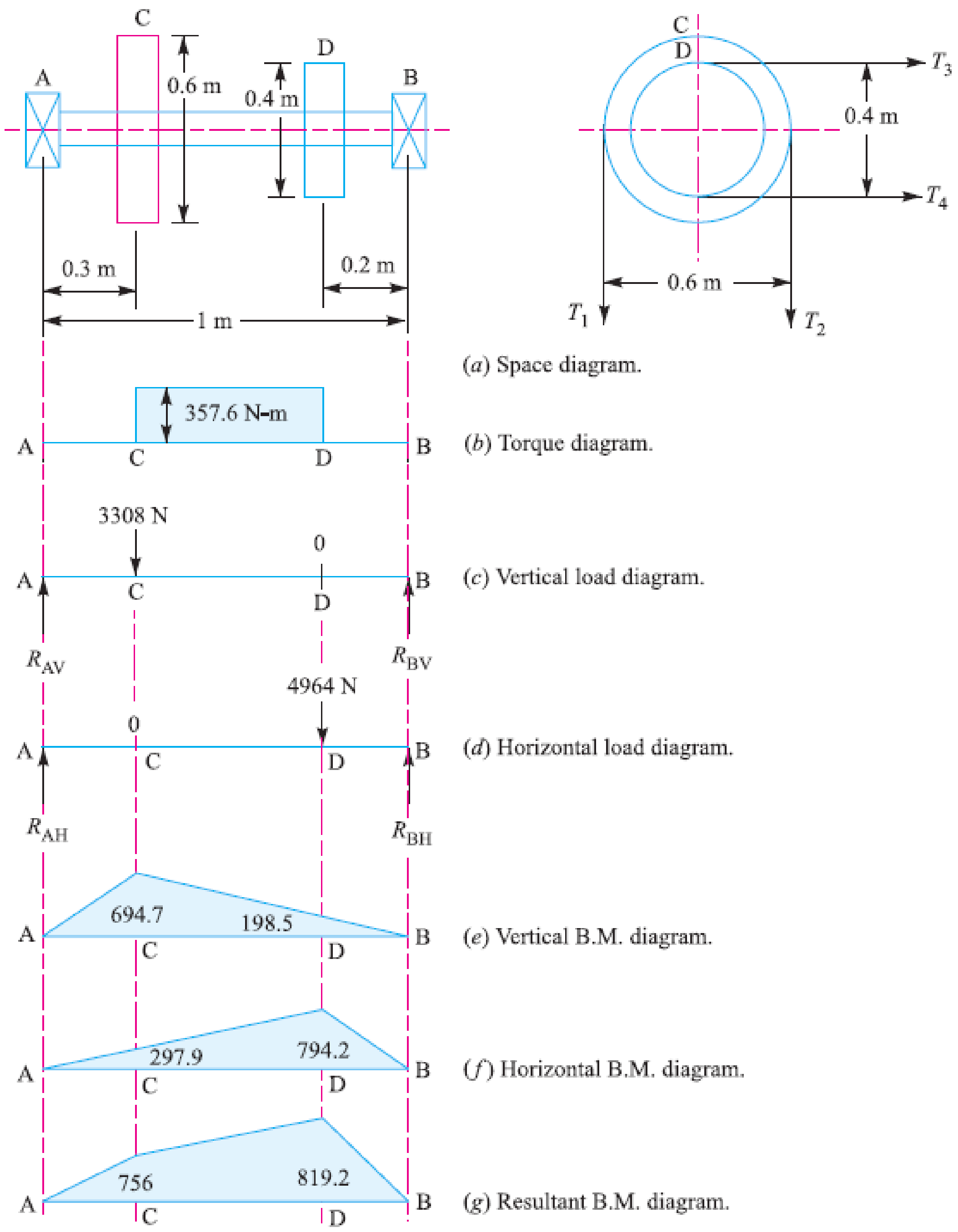


Fig.8.9 – Space diagram, Load diagram and B. M. diagram of the shaft

Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

Vertical load on the shaft at D = 0

The vertical load diagram is shown in *Fig.8.9* (c).

We know that torque acting on the pulley C,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig. 6.18 (b).

Let T_3 = Tension in the tight side of the belt on pulley D, and

T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (i.e. C and D) is same, therefore

$(T_3 - T_4)\, R_D = T = 357.6$ N-m

$$(T_3 - T_4) = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788\ N \qquad \textit{Eq. (8.13)}$$

Also, $$\frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127$$

$$T_3 = 2.127\, T_4 \qquad \textit{Eq. (8.14)}$$

From *Eq. (8.13)* and *Eq. (8.14)*, we find that

$T_3 = 3376$ N, and $T_4 = 1588$ N

∴ Horizontal load acting on the shaft at D,

$W_D = T_3 + T_4 = 3376 + 1588 = 4964$ N

Also horizontal load on the shaft at C = 0

The horizontal load diagram is shown in *Fig.8.9* (d).

Now let us find the maximum bending moment for vertical and horizontal loading.

Considering the vertical loading at C, Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that,

$R_{AV} + R_{BV} = 3308$ N

Taking moments about A,

$R_{BV} \times 1 = 3308 \times 0.3$ or $R_{BV} = 992.4$ N

$R_{AV} = 3308 - 992.4 = 2315.6$ N

We know that B. M. at A and B,

$M_{AV} = M_{BV} = 0$

B. M. at C, $M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7$ N-m

B. M. at D, $M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5$ N-m

The bending moment diagram for vertical loading is shown in *Fig.8.9* (e).

Now considering horizontal loading at D, Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that,

$R_{AH} + R_{BH} = 4964$ N

Taking moments about A,

$R_{BH} \times 1 = 4964 \times 0.8$ or $R_{BH} = 3971$ N

$R_{AH} = 4964 - 3971 = 993$ N

We know that B. M. at A and B,

$M_{AH} = M_{BH} = 0$

B. M. at C, $M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9$ N-m

B. M. at D, $M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2$ N-m

The bending moment diagram for horizontal loading is shown in *Fig.8.9* (f).

Resultant B. M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \ N.m$$

Resultant B. M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \ N.m$$

The resultant bending moment diagram is shown in *Fig.8.9* (g).

We see that bending moment is maximum at D.

∴ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(M)^2 + (T)^2} = \sqrt{(819.2)^2 + (357.6)^2}$$

$$= 894 \ N.m \ = 894 \times 10^3 \ N.mm$$

We also know that equivalent twisting moment (T_e),

$$T_e = \frac{\pi}{16} \times \tau \times d^3$$

$$894 \times 10^3 = \frac{\pi}{16} \times 42 \times d^3$$

$$\boldsymbol{d = 47.6 \ mm}$$

Again we know that equivalent bending moment,

$$M_e = \left[\frac{1}{2}\left(M + \sqrt{(M)^2 + (T)^2}\right)\right] = \frac{1}{2}(M + T_e)$$

$$M_e = \frac{1}{2}(819.2 + 894) = 856.6 \ N.m = 856.6 \times 10^3 \ N.mm$$

We also know that equivalent bending moment (M_e),

$$M = \frac{\pi}{32} \times \sigma_b \times d^3$$

$$856.6 \times 10^3 = \frac{\pi}{32} \times 63 \times d^3$$

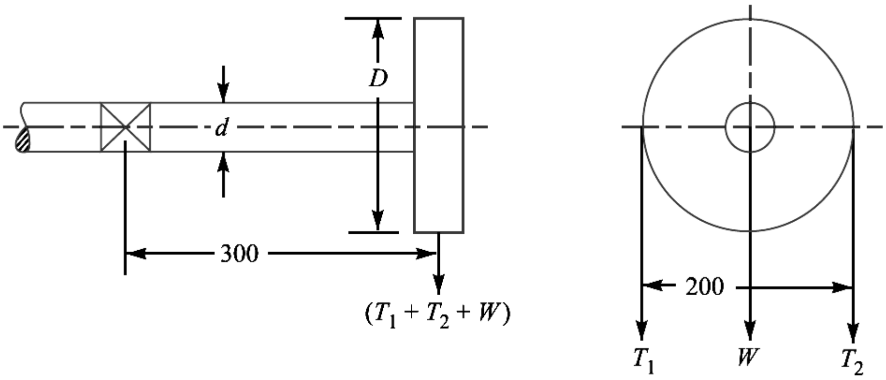
$$\boldsymbol{d = 51.7 \ mm}$$

Taking larger of the two values, we have d = 51.7 say 55 mm

Ex. 8.4 Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 r.p.m. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively. The allowable shear stress in the shaft may be taken as 35 MPa.

Solution:

Given Data:	***Given Data:***	***To be Calculated:***
W = 200 N;	θ = 180° = π Radian;	$d = ?$
L = 300 mm;	μ = 0.3;	
D = 200 mm or R = 100 mm;	K_m = 1.5;	
P = 1 kW = 1000 W;	K_t = 2;	
N = 120 r.p.m.	τ = 35 MPa = 35 N/mm^2	



We know that torque transmitted by the shaft,

$$T = \frac{60\,P}{2\pi N} = \frac{60\ \times 1000}{2\pi \times 120}$$

$$= 79.6\ N.m = 79600\ N.mm$$

We know that torque transmitted by the shaft,

$$T = (T_1 - T_2)R$$

$$79600 = (T_1 - T_2)100$$

$$(T_1 - T_2) = 796\ N \qquad \textit{Eq. (8.15)}$$

We know that

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{(0.3\times\pi)} = 2.57 \qquad \textit{Eq. (8.16)}$$

From *Eq. (8.15)* and *Eq. (8.16)*, we get,

T_1 = 1303 N, and T_2 = 507 N

We know that the total vertical load acting on the pulley,

$W_T = T_1 + T_2 + W$ = 1303 + 507 + 200 = 2010 N

Bending moment acting on the shaft,

M = W_T × L = 2010 × 300 = 603 × 10^3 N-mm

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_m \times T)^2}$$

$$T_e = \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3\ \ N.mm$$

We also know that equivalent twisting moment (T_e),

$$T_e = \frac{\pi}{16} \times \tau \times d^3$$

$$918 \times 10^3 = \frac{\pi}{16} \times 35 \times d^3$$

$$\boldsymbol{d = 51.1 \cong 55\ mm}$$

Ex. 8.5 A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.

Solution:

Given Data:

P = 4 kW

N = 800 rpm

Given Data:

G = 84 GPa

θ = 0.25°

To be Calculated:

a) $d = ?$

b) $\tau = ?$

θ = 0.25° = $0.25 \times \frac{\pi}{180}$ = 0.0044 Radian

Diameter of the spindle

The torque transmitted by the shaft,

$$T = \frac{60\,P}{2\pi N} = \frac{60\ \times 4000}{2\pi \times 800} = 47.74\ N.m = 47740\ N.mm$$

We also know that

$$\frac{T}{J} = \frac{G.\theta}{L}$$

$$J = \frac{T.L}{G.\theta}$$

$$\frac{\pi}{32} \times d^4 = \frac{47740 \times 1000}{84000 \times 0.0044} = 129167$$

$$\boldsymbol{d = 33.87 \cong 35\ mm}$$

Shear stress induced in the spindle

We know that the torque transmitted by the spindle (T),

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$47740 = \frac{\pi}{16} \times \tau \times 35^3$$

$$\tau = 5.67\ MPa$$

Ex. 8.6 Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft is being half the external diameter. Both the shafts have the same material and length.

Solution: ***Given Data:***

$d_o = d$;

$$d_i = \frac{d_O}{2}$$

$$k = \frac{d_i}{d_O} = 0.5$$

Comparison of weight

We know that weight of a hollow shaft,

W_H = Cross-sectional area × Length × Density

$$W_H = \frac{\pi}{4}[(d_O)^2 - (d_i)^2] \times Length \times Density \qquad \textit{Eq. (8.17)}$$

Also weight of the solid shaft,

$$W_S = \frac{\pi}{4}(d)^2 \times Length \times Density \qquad \textit{Eq. (8.18)}$$

Since both the shafts have the same material and length, therefore by dividing *Eq. (8.17)* by *Eq. (8.18)*, we get

$$\frac{W_H}{W_S} = \frac{(d_O)^2 - (d_i)^2}{(d)^2} = \frac{(d_O)^2 - (d_i)^2}{(d_O)^2}$$

$$\frac{W_H}{W_S} = 1 - \frac{(d_i)^2}{(d_O)^2} = 1 - k^2 = 1 - (0.5)^2$$

$$\frac{W_H}{W_S} = 0.75$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau \times (d_O)^3 [1 - k^4] \quad \textit{Eq. (8.19)}$$

Also strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times (d)^3 \quad \textit{Eq. (8.20)}$$

Dividing *Eq. (8.19)* by *Eq. (8.20)*, we get

$$\frac{T_H}{T_S} = \frac{(d_O)^3[1 - k^4]}{(d)^3} = \frac{(d_O)^3[1 - k^4]}{(d_O)^3} = [1 - k^4]$$

$$\frac{T_H}{T_S} = [1 - (0.5)^4] = 0.9375$$

Comparison of stiffness

We know that stiffness,

$$S = \frac{T}{\theta} = \frac{G.J}{L}$$

∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times J_H$$

$$S_H = \frac{G}{L} \times \frac{\pi}{32} [(d_O)^4 - (d_i)^4] \quad \textit{Eq. (8.21)}$$

Also stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times J_S$$

$$S_S = \frac{G}{L} \times \frac{\pi}{32} (d)^4 \quad \textit{Eq. (8.22)}$$

Dividing *Eq. (8.21)* by *Eq. (8.22)*, we get

$$\frac{S_H}{S_S} = \frac{(d_O)^4 - (d_i)^4}{(d)^4} = \frac{(d_O)^4 - (d_i)^4}{(d_O)^4}$$

$$\frac{S_H}{S_S} = 1 - \frac{(d_i)^4}{(d_O)^4} = 1 - k^4 = [1 - (0.5)^4]$$

$$\frac{S_H}{S_S} = 0.9375$$

Ex. 8.7 A 45 mm diameter shaft is made of steel with yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Solution:

Given Data:
d = 45 mm;
w = 14 mm;
t = 9 mm;

Given Data:
for shaft, σ_{yt} = 400 MPa
For Key, σ_{yt} = 340 MPa
F.S. = 2

To be Calculated:
a) $l = ?$

Let *l* = Length of key.

According to maximum shear stress theory, the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100\ N/mm^2$$

Also maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85\ N/mm^2$$

Maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3$$

$$T = \frac{\pi}{16} \times 100 \times 45^3 = 1.8 \times 10^6\ \ N.mm$$

Let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$T = l \times w \times \tau_k \times \frac{d}{2}$$

$$1.8 \times 10^6 = l \times 14 \times 85 \times \frac{45}{2}$$

$$l = 67.2\ mm$$

Now let us consider the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$T = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$1.8 \times 10^6 = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} \qquad \left(Tacking\ \ \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$$l = 104.6\ mm$$

Taking the larger of the two values, we have

$$\boldsymbol{l = 104.6\ \approx 105\ mm}$$

8.5 References

1) A Textbook of Machine Design by R.S. Khurmi, S. Chand Publication.
2) Strength of Materials by R.S. Khurmi, S. Chand Publication.
3) Design of Machine Elements by V.B. Bhandari, McGraw-Hill Publication.