7

Network Analysis

Contents

7.1 Introduction

Every management has limited resources both human and non-human. These resources are provided to the project managers for completing the project with constraints of the specified time. So, each manager has a task to complete the project successfully as well as within time. This leads the manager to have good project management approach.

Project management involves project planning, project scheduling and project controlling. Project planning and project scheduling are the two steps that must be performed before the starting of the project and project controlling starts as the execution of the project is done.

Project planning involves the identification of different tasks that are necessary to complete the project and the identification of resources required for completing those tasks. These resources are men, material, machine and money. Project planning involves estimation of time for each task.

Project scheduling involves the sequencing of the project tasks along the given time frame. Scheduling also involves the computation of resources required at a particular time. It also includes identification of tasks that are critical and limited, which may affect the timeline of the project.

Project controlling starts after the completion of the above two important tasks. The controlling phase keeps an eye on the scheduled time of completion of project and actual time of completion of the project. In other words, we can say that controlling phase finds the deviations in actual progress.

To have efficient project management different tools are required to be implemented. A network analysis is one of the important and most widely used tool for project management.

A network is a symbolic representation of the essential characteristics of the project. CPM (Critical Path Method) and PERT (Programme Evaluation and Review Technique) are two most widely used techniques as network techniques.

7.2 Terms Used in Network Analysis

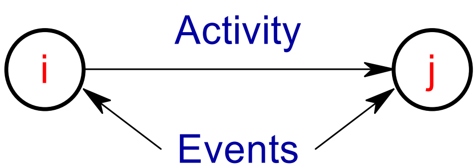
Following are different terms used in network analysis:

1. **Activity:** It is a physically identifiable part of the project which consumes time as well as resources for its execution. It is represented by an arrow. The tail represents start of that activity and head represents the finish of the activity. This arrow should be kept straight.

   Activity is represented as ⟶

   Each activity is represented by one and only one single arrow. If an activity is further subdivided into segments, each segment is represented by a separate arrow.

2. **Event:** It is the representation of the beginning and finishing points of activity. An event does not consume any time. It is represented by circle (node). For an activity, the starting point is called i^{th} event and the finishing point is called j^{th} event.



3. **Path:** An unbroken chain of activity arrows connecting the initial event to some other event is called a path.

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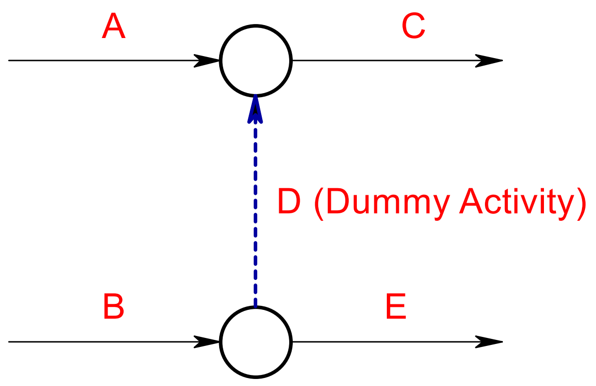
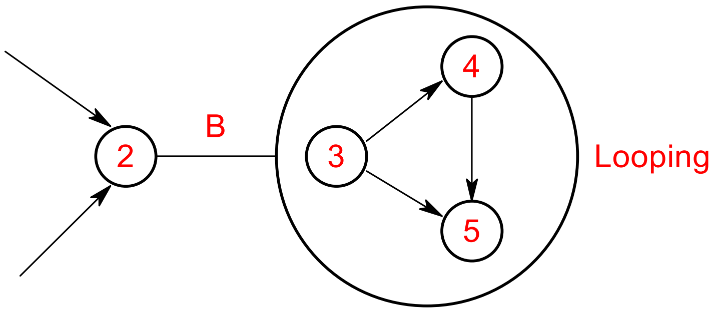
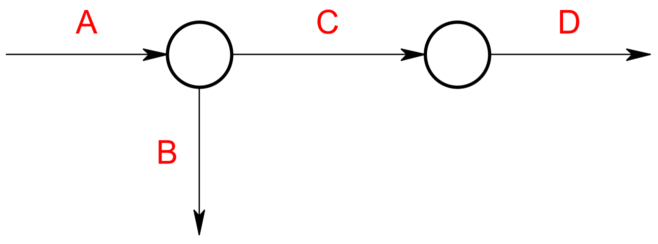
4. **Network Diagram:** It is the graphical representation of logically and sequentially connected arrows and nodes (representing activities and events) of the project.
5. **Predecessor activities:** In a constructed network diagram, the activity which is required to be completed before starting a particular activity is called predecessor activity.
6. **Successor Activities:** In a constructed network diagram, the activity which must follow any particular activity is called successor activity.
7. **Dummy Activity:** An activity which only shows the dependency of one activity on the other, but does not consume any resource is called dummy activity. Dummy activities are represented by dotted arrows. A dummy activity is an important activity when there is dependency of starting the one particular activity is on more than one activity. For example, if the activity C can only be started after the completion of activities A and B. Also, activity E depends only on the completion of activity B. So, at that time, it is necessary to introduce dummy activity as shown below:



8. **Looping of Activities:** Sometime in the network diagram, due to errors, the loop of different activities is formed. This is actually a mistake of considering the activities at the time of planning and this kind of loop must be avoided in the network diagram. The looping is as shown below:



9. **Dangling:** It is also an error formed due to the mistake in project planning. In this kind of error, any activity may be disconnected before the completion of all activities. As shown in figure below, activity B, is disconnected from the flow of the project.



7.3 Fulkerson's Rule

Once the network diagram is drawn in a logical sequence, every event is assigned a number, which is placed inside the circle. The sequence of numbers should be such that it represents the flow of the network. Following are the rules of D.R. Fulkerson for numbering purposes.

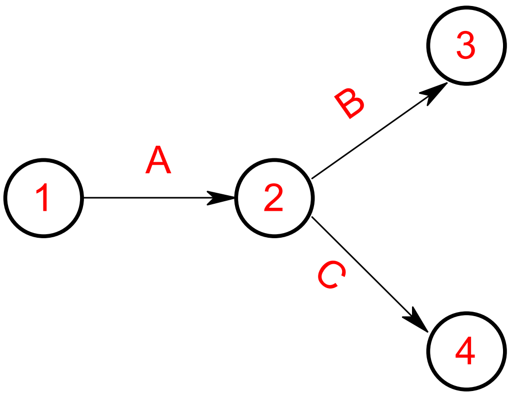
1. The event number should be unique.
2. Event number should be carried out in a sequential basis from left to right.
3. The initial event which has all outgoing arrows with no incoming arrow is numbered 0 or 1.
4. The head of an arrow should always bear a number higher than the one assigned at the tail of the arrow. For example – 1-2, 1-3, 2-4, 2-5, 3-7, 3-8, etc. and not 8-7 or 9-3 and the like.
5. Gaps should be left in the sequence of event numbering to accommodate subsequent inclusion of activities if necessary.

Network models are of two types:

a) Activity on Arrow (AOA)

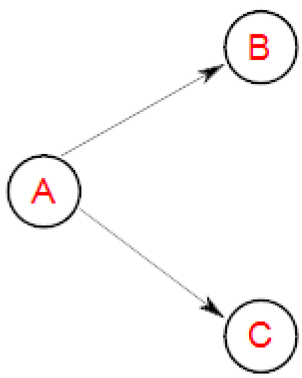
In this type of network model, each activity end at node (circle). These nodes represent point of starting or ending. Here, the arrow itself indicates the span of time required to complete the activity. This diagram starts with a single node with no predecessors may start. The diagram then follows from left to right, ending with a single node, where no followers come together. Dummy activities are also following the same treatment as real activity.

For example, if activity B and C must follow A activity, then following AOA diagram can be drawn:



b) Activity on Node (AON)

As mentioned in the previous section, a network diagram is also called, Arrow Diagram. So, if we introduce dummy activities in the diagram, we have to number them as events and that activity is required to be included to represent the precedence relationship between the activities to maintain logical relationship. So, this results in increased number of activities, lengthy and cumbersome networks and more time and effort for analysis.



To avoid above difficulty, activity is represented on node connected to the precedent activity. This kind of diagram is called Activity on Node (AON) diagrams. In the diagram, the tail of each arrow is on the predecessor activity, while the head is on the successor activity. The activity is indicated within the node. While the arrows show only the sequencing. If we take the same case of AOA diagram, then AON diagram of this is shown here.

7.4 Network Diagram (Steps & Important Points)

Following are the steps to be performed for preparation of network diagram:

1. Decide the number of activities.
2. Decide the order of the activities in a logical sequence.
3. Fix predecessor activities and successor activities.
4. Also, find out which activities can be done in parallel with other activities.

Some important points are to be taken care while drawing network diagram are as follows:

1. Arrows should not cross each other. Where crossing is not possible to avoid, bridging should be done.
2. No two or more activities can have the same tail and head events.
3. An event is not finished until all the activities flowing into it are completed.
4. No subsequent activity can begin until its tail event is completed.
5. Only one initial event and one end event is to be there in the network diagram.

7.5 Critical Path Method (CPM)

Each activity of the project takes some time for completion. If the time of any activity can be estimated perfectly then this activity is called *"Deterministic activity".* But, if any activity is performed the first time, the uncertainty is high, so the pre-assumed time may vary on any side. These activities are called *"variable activities".*

For evaluating the network having all activities as "deterministic activities", the technique used is called **"Critical Path Method (CPM)"**.

For analyzing the network diagram having "variable activities", the technique used is called **"Project Evaluation and Review Technique (PERT)"**.

In this section, we will work on CPM and in the next section, the concept of PERT is discussed.

Following are the steps for CPM:

Step 1: Calculate the time schedule for each activity

In this step, the determination of the starting time by which the activity must begin and finishing time at which the activity must be finished is determined. From that we can find out the earliest start time, earliest finish time, the latest start time and the latest finish time is determined.

Earliest start time (EST) is the time at which an activity can begin at the earliest. For EST it is assumed that all preceding activities are finished before starting that activity. Sometimes, the activity is depending on more than one activity, so until and unless both preceding activities are not completed, that depending activity cannot be started. Following is the way to calculate EST.

If only one activity converges on an event, its EST is given by EST's of the tail event of the activity added with activity duration. If more than one activity converges on it, EST's via all the paths would be calculated and the highest value is chosen and termed as EST.

Latest Finish Time (LFT) is the time by which the activity must be finished and the project cannot be delayed more than that. This is calculated by, proceeding progressively from the end event to the start event. The LFT for the last event is assumed to be equal to its EST. LFT's for other events are calculated by the following rules.

If only one activity branches from an event, then compute LFT by subtracting activity duration from the LFT of its head event. If more than one activity branches out from on event, then compute LFT's via all the paths and the lowest value is chosen and is termed as LFT.

Step 2: Calculate EFT and LST are calculated in the following way

$$EFT = EST + Time\ of\ the\ activity$$

$$LST = LFT - TIME\ of\ the\ activity$$

Step 3: Calculate the float in the various activities of the project

The float is determined based on EFT and LFT (or EST and LST). It is also termed as total float. It is the positive difference between the finish times or starting times.

$$Float\ (total\ float) = LFT - EFT\ (or\ LST - EST)$$

The total time is calculated after considering the sequence of the project and the time required to complete each activity.

Step 4: Identify the critical activities and find the critical path

Critical activities are those activities that must be started on time and must be completed on time. Otherwise the project may get delayed. Those activities are critical activities and the path through those activities is called critical path. That path is the longest path of the project. The other activities are considered non-critical activities.

7.5.1 Other Important Terminologies:-

a) Types of floats

Following are the types of floats:-

1. **Total Float:** The total float of an activity represents the amount of time by which it can be delayed without delaying the project completion date. In other words, it refers to the amount of free time associated with an activity which can be used before, during or after the performance of this activity.
   It is the difference between the start times (or Finish times).
   $$Total\ Float\ (TF) = (LFT - EFT)\ or\ (LST - EST)$$
2. **Free Float:** It is the part of total float within which an activity can be manipulated without affecting the floats of subsequent activities. It is calculated by subtracting the head event slack from the total Float.
   $$Free\ Float\ of\ i-j\ activity\ (FF) = TF - Head\ Event\ Slack$$
   where, $Head\ Event\ Slack = (LFT - EST)\ of\ the\ head\ event$
   Thus, we can say that free float is the time by which completion of an activity can be delayed without delaying its immediate successor activities.
3. **Interfering Float:** It is that part of the total float which causes a reduction in the floats of the succeeding activities. This float is that portion of activity float which cannot be consumed without adversely affecting the floats of the subsequent activities. It is calculated by subtracting free Float from the total float of the activity.
   $$Interfering\ Float\ of\ ith\ activity\ (IF) = TF - FF$$
4. **Independent Float:** It is that part of the total float within which an activity can be delayed for start without affecting the floats of preceding activities. It is calculated by subtracting the tail event slack from the free float.
   So, Independent Float of i-j activity (IF) = FF - (LFT - EST) of the tail event

Thus, we can say that independent float is the time by which starting of an activity can be delayed without delaying its immediate preceding activities.

b) Super critical Activity: An activity having negative float is called super critical activity. Such an activity demands very special attention and action. This means that the time available is less than actual time demands for its completion. So, management has to take decision on how to compress that time.

c) Sub critical Activity: An activity having next higher float than the critical activity. Such activity provides freedom of action. The network may have more than one sub critical path.

d) Slack: It is the difference between LFT and EST of the event" It is interpreted as the time by which occurrence of an event can be delayed. So,

S = (LFT - EST) of the event.

7.6 Programme Evaluation and Review Technique (PERT)

PERT is the technique used for "variable activities". In PERT, the time is combination of three different time estimations. PERT system is based on usually β probability distribution. Following are the three different time estimations:

1. The Optimistic Time estimate (t_o): The minimum time required for completion of the activity as per the predetermined conditions. No delays or setbacks are considered for this time estimation.
2. The Pessimistic Time estimate (t_p): The maximum time that activity will take under worst conditions. Here major catastrophes like floods, earthquakes, storms, etc. are not considered.
3. The Most Likely Time estimate (t_m): The time an activity will take if executed under normal condition.

For time estimation expert opinion is taken having their experience in the relevant field.

7.6.1 Important Terms in PERT Analysis

1. Expected time or average time (t_e): Since there are three-time values available in PERT, average time is to be calculated. The following formula is used in PERT analysis:

$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$

2. Variance (V): Variance is given by the following formula:

$$V = \left(\frac{t_p - t_o}{6}\right)^2$$

3. Standard Deviation (σ): It is the square root of the summation of activities variance.

$$\sigma = \sqrt{V_1^2 + V_2^2 + V^2{}_3\ldots\ldots}$$

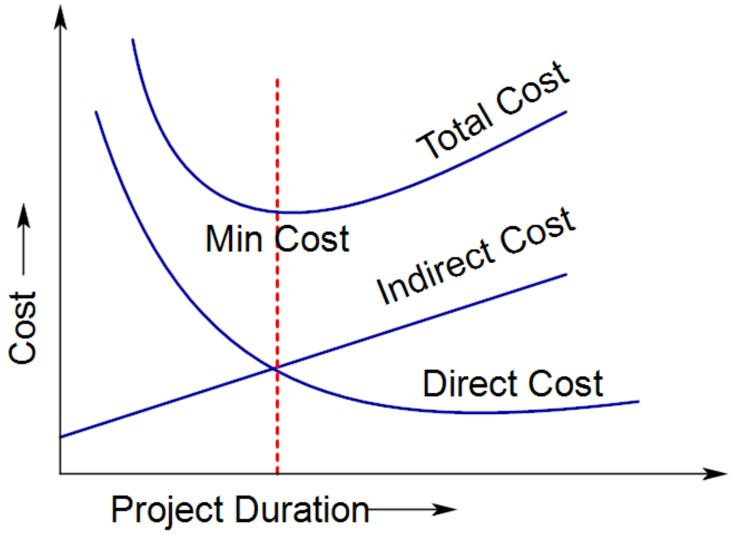
4. Probability of completion of the project (z): It is calculated in order to estimate that how many percentages are the chances of completion of the project in certain time (*t*). Here, t_{cp} is the time for completion of project on critical path and *t* is any certain time, then probability of completion of project in that time *t*, is given by,

$$Z = \frac{t - t_{cp}}{\sigma}$$

7.7 Crashing of Network

Each activity of the project consumes resources. So, the project manager has to calculate the project cost and it is required that project cost must be within the budget. This total project cost includes two types of costs: Direct cost and indirect cost.

- Direct cost is that type of cost which depends on the resources involved in the project. These resources are men, machines and materials included in the project.
- On the other hand, some indirect is involved which includes fixed cost and variable cost. Fixed costs such as license fees, rent, general and administrative expenses, etc. which is irrelevant of the progress of the project. Variable costs such as interest on capital, depreciation, etc. are depending on the time consumed by the project.



- he sum of both direct and indirect costs gives total project cost.

The total project cost depends on the time of the project. Direct cost is high at the beginning of the project due to purchase of the material and machines. So, as time goes on the direct cost decreases. Since indirect cost is depending on the time, it increases with time. When the total project cost curve is drawn, the total cost will have minimum value at a certain point of time and that point of time is called optimum time and that corresponding cost is called optimum cost.

Crashing of Network:

Now, in many cases, it is necessary to complete the project earlier than the duration of the time calculated based on CPM. When the project time is required to be reduced from Critical time, then it will cause more cost to the company. If we say, the time associated with CPM as normal time (t_n) and cost associated with CPM as (C_n), then the normal time (t_n) will reduce to crashed time (t_c), when it is required to finish the project early. This early finish will increase the cost of project from normal project cost (C_n) to the crashed cost (C_c). This process of time reduction is called the crashing of the project. Based on above data cost slope is calculated:

$$Cost\ Slope = \frac{Crash\ Cost - Normal\ Cost}{(Normal\ Time - Crash\ Time)} = \frac{C_c - C_n}{(t_n - t_c)}$$

It is important to select that critical activity for crashing which has minimum cost slope. Cost Slope means the ratio of increase in cost due to crashing to the decrease in time due to crashing. So, for selecting the crashing, the critical activity which has minimum cost slope is selected.

This crashing is carried out by reducing the normal time period of the critical activities of the project by incurring more cost than normal cost. If all the critical activities of the project are crashed, then the time is the minimum time or crashed time. This will increase the cost and that cost is called crashing cost of the project.

If all the activities of the project are crashed then, it will cost more and more cost, but it will not give an additional advantage over and above the one obtained by crashing only the critical activities. So, there is no need to go for crashing of noncritical activities.

However, in the process of crashing the critical activities, it may happen that some of the non-critical activities become critical. Therefore, it is essential to proceed step by step and crash each activity and check whether any other non-critical activity has become critical.

Following are the steps for crashing of the network:

1. Determine the normal project completion time and associated critical path.
2. Find out the cost slope for all activities.
3. For reducing the total project completion time, identify and crash an activity time having the lowest cost slope. This lowest-cost slope activity which is crashed is to be crashed until any one of the following stages is reached.
    i) Either the other critical path is generated or
    ii) That crashing activity is reached the maximum possibility of crashing.
4. If the critical path under crashing determined in step 1, then continue crashing the same path. Now, crash the next activity of the critical path having lowest cost slope.

Stop this procedure when each critical activity has been crashed to its lower possible time.

7.8 Comparison between CPM and PERT

As discussed earlier that though CPM and PERT networks were developed independently and the procedures adopted for the two techniques were, more or less common, the objectives were however different. The basic comparison between the two techniques is summarized below.

Table 7.1 - Comparison between CPM and PERT

CPM	PERT
A deterministic model with well-known activity times based upon past experience. It, therefore, does not deal with uncertainty in time.	A probability model with uncertainty in activity duration. The duration of each activity is normally computed from multiple time estimates with a view to taking into account time uncertainty. These estimates are ultimately used to arrive at the probability of achieving any given scheduled date of project completion.
It is activity-oriented as the results of calculations are considered in terms of activities or operations of the project.	It is said to be event-oriented as the results of an analysis are expressed in terms of events or distinct points in time indicative of progress.
The use of dummy activities is not necessary.	The use of dummy activity is required for representing the proper sequencing.
CPM is commonly used for those projects which are repetitive in nature and where one has prior experience of handling similar projects.	PERT is generally used for those projects where time required to complete various activities is not known as precisely.
It is used for construction projects and business problems.	It is applied for widely planning and scheduling, research programmes and developing projects.

CPM deals with the costs of project schedules and their minimization. The concept of crashing is applied mainly to CPM models.	PERT analysis does not usually consider costs.
It is difficult to use CPM as a controlling device for the simple reason that one must repeat the entire evaluation of the project each time, the changes are introduced into the network.	PERT is an important control device too, for it assists the management in controlling a project by calling attention as a result of constant review to such delays in activities which might cause a delay in the project's completion date.
CPM places dual emphasis on time cost and evaluates the trade-off between project cost and time. By deploying additional resources, it allows the project manager to manipulate project duration within certain limits so that project duration can be shortened at an optimal cost.	PERT helps the manager to schedule and coordinate various activities so that the project can be completed on schedule time.

7.9 Class Examples

Ex. 7.1 Draw a network corresponding to the following information.

Activity	Predecessors	Description
A	-	Locate facility
B	A	Order furniture
C	F	Interview
D	-	Hire & Train
E	A	Remodel
F	B	Furniture setup
G	D,E,F	Move in

Solution:

Ex. 7.2 A project consists of tasks labelled A, B, ..., H, I with the following relationships (W < X, Y means X and Y cannot start until W is completed: X, Y < means W cannot start until both X and Y are completed). With this notation construct the network diagram having the following constraints:

$$A < D, E; B, D < F; C < G; B < H; F, G < I$$

Solution:

Ex. 7.3 Draw a network corresponding to the following information.

Activity	Predecessors	Description
A	None	Obtain building permits
B	A	Build concrete forms
C	A	Excavate pool area
D	B and C	Pour and cure concrete
E	None	Install above-ground filter pump
F	E	Install electrical systems
G	None	Obtain building permits

Solution:

Ex. 7.4 a) Draw a network corresponding to the following information & determine scheduling times and floats.

Activity	**Time (days)**	**Activity**	**Time (days)**
1 - 2	8	6 – 8	10
1 – 3	2	7 – 10	12
1 – 4	6	8 – 9	3
1 – 5	12	8 – 10	6
2 – 4	5	9 – 12	8
2 – 7	9	10 – 12	18
3 – 5	3	10 – 14	9
3 – 6	7	11 – 12	7
4 – 10	4	11 - 14	4
5 – 11	10	12 – 13	11
6 - 7	2	13 - 14	4

b) Find the earliest and latest scheduling times of various activity.

c) Obtain the total interference float and independent float for each of the activity.

d) Can this project be complete within 65 days?

e) What would be the effect on the project length of reducing the resource to be used for activity 8-10 by such an amount as would increase the time for this activity by 5 days?

f) The HOD in which activity 6-8 is to be performed requests that the allowed to work overtime so that the activity can be completed in 6 days. Should his request be considered in the interest of project completion at an earlier date? How about a similar request from the manager of activity 2-7?

g) It has come to be known that are to non-availability of resources in time activity 3-5 would be

    (1) The project completion time?

    (2) The start of its successor activity by how much?

Solution:

Ex. 7.5 The following table gives the list of various activities involves in launch of new CREDIT CARD service by a company their immediate predecessor & their expected duration (in days)

Activity	Immediate Predecessor	Expected Durations		
		t_o	t_m	t_p
A	-	10	12	14
B	A	14	15	17
C	B	2	3	4
D	C	4	6	8
E	C	10	12	14
F	B	20	25	27
G	C	10	17	20
H	F	5	6	7
I	D	7	12	14
J	H , I	14	17	20
K	C	1	2	3
L	K	10	15	20
M	L	3	5	7
N	M , J	13	15	17
O	N	20	21	22
P	O	7	9	14
Q	P	2	3	4
R	Q	2	2	2
S	P	7	10	13
T	S	5	7	9
U	T , R, G	4	8	12

(a) Draw an error diagram for project.

(b) Find the expected project completion time.

(c) Determine probability of the completing the project in 165 days.

(d) What is probability of completing in 155 days?

(e) If manager wants to 95% sure that about project completing time when we should start project?

Solution:

Ex. 7.6 The following table shows for each activity of a project the normal & crash times as also the normal & crash costs. The contract includes a penalty clause of Rs. 200 per day in excess of 19 days. The overhead cost in Rs. 400 per day.

(a) Draw the network for the project & determine the critical path.
(b) Find the cost of completing the project in normal time.
(c) Crash the project activities & determine the cost of completing the project in the minimum time.
(d) What is the optimal duration of the project & what is the cost involves?

Activity	Time (days)		Cost (Rs.)		Crashing cost/day (Rs.)
	Normal	Crash	Normal	Crash	
1 - 2	6	4	600	1000	200
1 - 3	4	2	600	1400	400
2 - 4	5	3	500	1500	500
2 - 5	3	1	450	650	100
3 - 4	6	4	900	2000	550
4 - 6	8	4	800	3000	550
5 - 6	4	2	400	1000	300
6 - 7	3	2	450	800	350

Solution: