

# 6

## Fundamentals of Compressible Flow

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## 6.1 Introduction

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- ▶ For most part, we have limited our consideration so far to flows for which density variations and thus compressibility effects are negligible.
- ▶ In this chapter we lift this limitation and consider flows that involve significant changes in density. Such flows are called *compressible flows*, and they are frequently encountered in devices that involve the flow of gases at very high speeds.
- ▶ Compressible flow combines fluid dynamics and thermodynamics in that both are absolutely necessary to the development of the required theoretical background.
- ▶ “A **compressible flow** is that flow in which the density of the fluid changes during flow.”
- ▶ All real fluids are compressible to some extent and therefore their density will change with change in pressure or temperature. If the relative change in density  $\Delta\rho/\rho$  is small, the fluid can be treated as incompressible.
- ▶ A compressible fluid, such as air, can be considered as incompressible with constant  $\rho$  if changes in elevation are small, acceleration is small, and/or temperature changes are negligible.
- ▶ In other words, if Mach number ( $M = V/C$ ), where  $C$  is the sonic velocity, is small, compressible fluid can be treated as incompressible.
- ▶ The gases are treated as compressible fluids and study of this type of flow is often referred to as ‘Gas dynamics’.
- ▶ Some important problems where compressibility effect has to be considered are:
  - (i) Flow of gases through nozzles, orifices
  - (ii) Compressors
  - (iii) Flight of aeroplanes and projectiles moving at higher altitudes
  - (iv) Water hammer and acoustics
- ▶ Compressibility affects the drag coefficients of bodies by formation of shock waves, discharge coefficients of measuring devices such as orificemeters, venturimeters and pitot tubes, stagnation pressure and flows in converging-diverging sections.

## 6.2 Adiabatic Energy Equation

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- ▶ As the flow of compressible fluid is steady, the Euler equation is given as :

$$\frac{dp}{\rho} + VdV + gdZ = 0 \quad \text{Eq. (6.1)}$$

- ▶ Integrating both sides, we get

$$\int \frac{dp}{\rho} + \int VdV + \int gdZ = \text{constant}$$

- ▶ In compressible flow since  $\rho$  is not constant it cannot be taken outside the integration sign. In compressible fluids the pressure ( $p$ ) changes with change of density ( $\rho$ ), depending on the type of process. In case of an adiabatic process,

$$pv^\gamma = \text{constant} \text{ or } \frac{p}{\rho^\gamma} = \text{constant} = c_2$$

$$\therefore \rho^\gamma = \frac{p}{c_2} \text{ or } \rho = \left(\frac{p}{c_2}\right)^{1/\gamma}$$

▶ Hence  $\int \frac{dp}{\rho} = \int \frac{dp}{\left(\frac{p}{c_2}\right)^{1/\gamma}} = (c_2)^{1/\gamma} \int \frac{1}{p^{1/\gamma}} dp = (c_2)^{1/\gamma} \int p^{-1/\gamma} dp$

$$\begin{aligned} &= (c_2)^{1/\gamma} \left[ \frac{p^{-\frac{1}{\gamma}+1}}{\left(-\frac{1}{\gamma}+1\right)} \right] = \frac{(c_2)^{1/\gamma} p^{\left(\frac{\gamma-1}{\gamma}\right)}}{\left(\frac{\gamma-1}{\gamma}\right)} = \frac{\gamma}{\gamma-1} (c_2)^{1/\gamma} p^{\left(\frac{\gamma-1}{\gamma}\right)} \\ &= \left(\frac{\gamma}{\gamma-1}\right) \left(\frac{p}{\rho^\gamma}\right)^{1/\gamma} p^{\left(\frac{\gamma-1}{\gamma}\right)} \quad (\because c_2 = \frac{p}{\rho^\gamma}) \\ &= \left(\frac{\gamma}{\gamma-1}\right) \frac{p^{\left(\frac{1}{\gamma}+\frac{\gamma-1}{\gamma}\right)}}{\rho} = \left(\frac{\gamma}{\gamma-1}\right) \frac{p}{\rho} \end{aligned}$$

▶ Substituting the value of  $\int \frac{dp}{\rho}$

$$\left(\frac{\gamma}{\gamma-1}\right) \frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

▶ Dividing both sides by g, we get

$$\left(\frac{\gamma}{\gamma-1}\right) \frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \quad \text{Eq. (6.2)}$$

▶ Above equation is called the Bernoulli's equation or energy equation for compressible flow undergoing adiabatic process.

### 6.3 Mach Number And Its Significance

- ▶ The Mach number is an important parameter in dealing with the flow of compressible fluids, when elastic forces become important and predominant.
- ▶ "Mach number is defined as the square root of the ratio of the inertia force of a fluid to the elastic force."
- ▶ Mach number is given by,

$$\begin{aligned} M &= \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho AV^2}{KA}} = \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}} \quad (\because \sqrt{K/\rho} = C) \\ M &= \frac{V}{C} = \frac{\text{Velocity at a point in a fluid}}{\text{Velocity of sound at that point at a given instant of time}} \quad \text{Eq. (6.3)} \end{aligned}$$

▶ Depending on the value of Mach number, the flow can be classified in various regions of flow as follows :

1. Subsonic flow : Mach number is less than 1.0 (or  $M < 1$ ) ; in this case  $V < C$ .
2. Sonic flow : Mach number is equal to 1.0 (or  $M = 1$ ) ; in this case  $V = C$ .
3. Supersonic flow : Mach number is greater than 1.0 (or  $M > 1$ ) ; in this case  $V > C$ .
4. Transonic flow : : Mach number is slightly less to slightly greater than 1.0 (or  $M \cong 1$ )
5. Hypersonic flow: Mach number,  $M \gg 1$ .

- ▶ The following points are worth noting :

(i) Mach number is important in those problems in which the flow velocity is comparable with the sonic velocity (velocity of sound). It may happen in case of airplanes travelling at very high speed, projectiles, bullets etc.

(ii) If for any flow system the Mach number is less than about 0.4, the effects of compressibility may be neglected (for that flow system).

## 6.4 Velocity of Sound

The solids as well as fluids consist of molecules. Whereas the molecules in solids are close together, these are relatively apart in fluids.

Consequently whenever there is a minor disturbance, it travels instantaneously in case of solids ; but in case of fluid the molecules change its position before the transmission or propagation of the disturbance.

Thus the velocity of disturbance in case of fluids will be less than the velocity of the disturbance in solids. In case of fluid, the propagation of disturbance depends upon its elastic properties. The velocity of disturbance depends upon the changes in pressure and density of the fluid.

The propagation of disturbance is similar to the propagation of sound through a media. The speed of propagation of sound in a media is known as acoustic or **sonic velocity** and depends upon the difference of pressure.

### 6.4.1 Derivation Of Sonic Velocity (Velocity Of Sound)

- ▶ Consider a one-dimensional flow through long straight rigid pipe of uniform cross-sectional area filled with a frictionless piston at one end as shown in Fig. 6.1.
- ▶ The tube is filled with a compressible fluid initially at rest. If the piston is moved suddenly to the right with a velocity, a pressure wave would be propagated through the fluid with the velocity of sound wave.

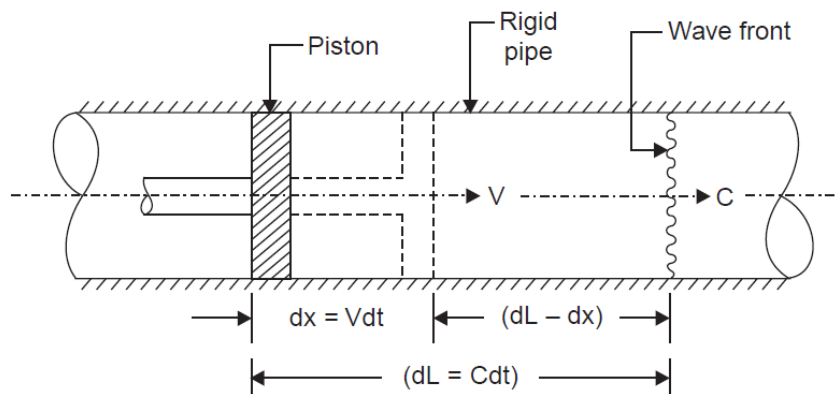


Fig.6.1 – One dimensional pressure wave propagation

- Let,  $A$  = cross-sectional area of the pipe,  
 $V$  = piston velocity,  
 $p$  = fluid pressure in the pipe before the piston movement,  
 $\rho$  = fluid density before the piston movement,  
 $dt$  = a small interval of time during which piston moves, and  
 $C$  = velocity of pressure wave or sound wave (travelling in the fluid).

- ▶ Before the movement of the piston the length  $dL$  has an initial density  $\rho$ , and its total mass =  $\rho \times dL \times A$ .
- ▶ When the piston moves through a distance  $dx$ , the fluid density within the compressed region of length  $(dL - dx)$  will be increased and becomes  $(\rho + d\rho)$  and subsequently the total mass of fluid in the compressed region  $(\rho + d\rho)(dL - dx) \times A$ .
- ▶ By principle of continuity,  $\therefore \rho \times dL \times A = (\rho + d\rho)(dL - dx) \times A$

But,  $dL = C dt$  and  $dx = V dt$ ; therefore, the above equation becomes

$$\begin{aligned}\rho C dt &= (\rho + d\rho)(C - V) dt \\ \rho C &= (\rho + d\rho)(C - V) \text{ or } \rho C = \rho C - \rho V + d\rho \cdot C - d\rho \cdot V \\ 0 &= -\rho V + d\rho \cdot C - d\rho \cdot V\end{aligned}$$

Neglecting the term  $d\rho \cdot V$  ( $V$  being much smaller than  $C$ ), we get

$$\begin{aligned}d\rho \cdot C &= \rho V \\ C &= \frac{\rho V}{d\rho}\end{aligned}\tag{Eq. (6.4)}$$

- ▶ Further in the region of compressed fluid, the fluid particles have attained a velocity which is apparently equal to  $V$  (velocity of the piston), accompanied by an increase in pressure  $dp$  due to sudden motion of the piston. Applying impulse-momentum equation for the fluid in the compressed region during  $dt$ , we get

$$\begin{aligned}dp \times A \times dt &= \rho \times dL \times A (V - 0) \\ \text{force on the fluid)} &\quad \text{(rate of change of momentum)}\end{aligned}$$

$$\begin{aligned}dp &= \rho \frac{dL}{dt} V = \rho \times \frac{C dt}{dt} \times V = \rho C V \\ C &= \frac{dp}{\rho V}\end{aligned}\tag{Eq. (6.5)}$$

- ▶ Multiplying eqns. (6.4) and (6.5), we get

$$\begin{aligned}C^2 &= \frac{\rho V}{d\rho} \times \frac{dp}{\rho V} \\ C^2 &= \frac{dp}{d\rho} \\ C &= \sqrt{\frac{dp}{d\rho}}\end{aligned}\tag{Eq. (6.6)}$$

#### 6.4.2 Sonic Velocity in Terms of Bulk Modulus

- ▶ The bulk modulus of elasticity of fluid ( $K$ ) is defined as,

$$K = \frac{dp}{\left(\frac{dv}{v}\right)}\tag{Eq. (6.7)}$$

Where,  $dv$  = decrease in volume, and  $v$  = original volume

(- ve sign indicates that volume decreases with increase in pressure)

- ▶ Also, volume,  $v \propto \frac{1}{\rho}$  or  $v\rho = \text{constant}$

Differentiating both sides, we get

$$v d\rho + \rho dv = 0 \text{ or } -\frac{dv}{v} = \frac{d\rho}{\rho}$$

Substituting the value of  $-\frac{dv}{v}$  from eq. (6.7), we get

$$\frac{dp}{K} = \frac{d\rho}{\rho} \text{ or } \frac{dp}{d\rho} = \frac{K}{\rho} \quad \text{Eq. (6.8)}$$

Substituting this value of  $\frac{dp}{d\rho}$  in eq. (6.6), we get

$$C = \sqrt{\frac{K}{\rho}} \quad \text{Eq. (6.9)}$$

- ▶ This eq. (6.9) is applicable for liquids and gases.

## 6.5 Propagation Of Disturbance In Compressible Fluid (Mach Waves)

- ▶ When some disturbance is created in a compressible fluid (elastic or pressure waves are also generated), it is propagated in all directions with **sonic velocity** ( $C$ ) and its nature of propagation depends upon the Mach number ( $M$ ).
- ▶ Such disturbance may be created when an object moves in a relatively stationary compressible fluid or when a compressible fluid flows past a stationary object.
- ▶ Consider a tiny projectile moving in a straight line with **velocity** ( $V$ ) through a compressible fluid which is stationary.
- ▶ Let the projectile is at 'A' when time  $t = 0$ , then in time 't' it will move through a distance  $AB = Vt$ . During this time the disturbance which originated from the projectile when it was at 'A' will grow into the surface of sphere of radius ' $Ct$ ' as shown in Fig. 6.2, which also shows the growth of the other disturbances which will originate from the projectile at every 't/4' interval of time as the projectile moves from A to B.
- ▶ Let us find nature of propagation of the disturbance for different Mach numbers.

### Case I. When $M < 1$ (i.e., $V < C$ , Subsonic flow) :

- ▶ In this case since  $V < C$ , let take  $V = 1$  unit and  $C = 2$  units, so that  $\frac{V}{C} = \frac{1}{2}$  which is less than 1.0.
- ▶ Let the projectile is at 'A' moving towards right, in 4 seconds the projectile reaches to position 'B'. The position of the projectile after 1 sec, 2 sec, 3 sec, 4 sec along the lines are shown by the points 3, 2, 1 and B respectively.
- ▶ The projectile moves from A to B in 4 seconds and hence the distance  $AB = 4 \times V = 4 \times 1 = 4$  units. The disturbance created at A in 4 seconds will move a distance  $= 4 \times C = 4 \times 2 = 8$  units in all directions. Hence taking A as a centre and radius equal to 8 units, a circle is drawn. This circle gives position of disturbance after after 4 seconds. When the projectile is at point 3, it will reach B in three seconds and distance  $3B = 3 \times V = 3 \times 1 = 3$  units. But the disturbance created at point 3 in three seconds will move a distance having a radius  $= 3 \times C = 3 \times 2 = 6$  units. Similarly at point 2, the disturbance will have a radius  $= 2 \times C = 2 \times 2 = 4$  units and at point 1, the disturbance will have radius  $= 1 \times C = 1 \times 2 = 2$  units as shown in Fig. 6.2 (a).
- ▶ In this case, the projectile **lags behind** the disturbance/pressure wave and hence the projectile at point B lies inside the sphere of radius 8 units.

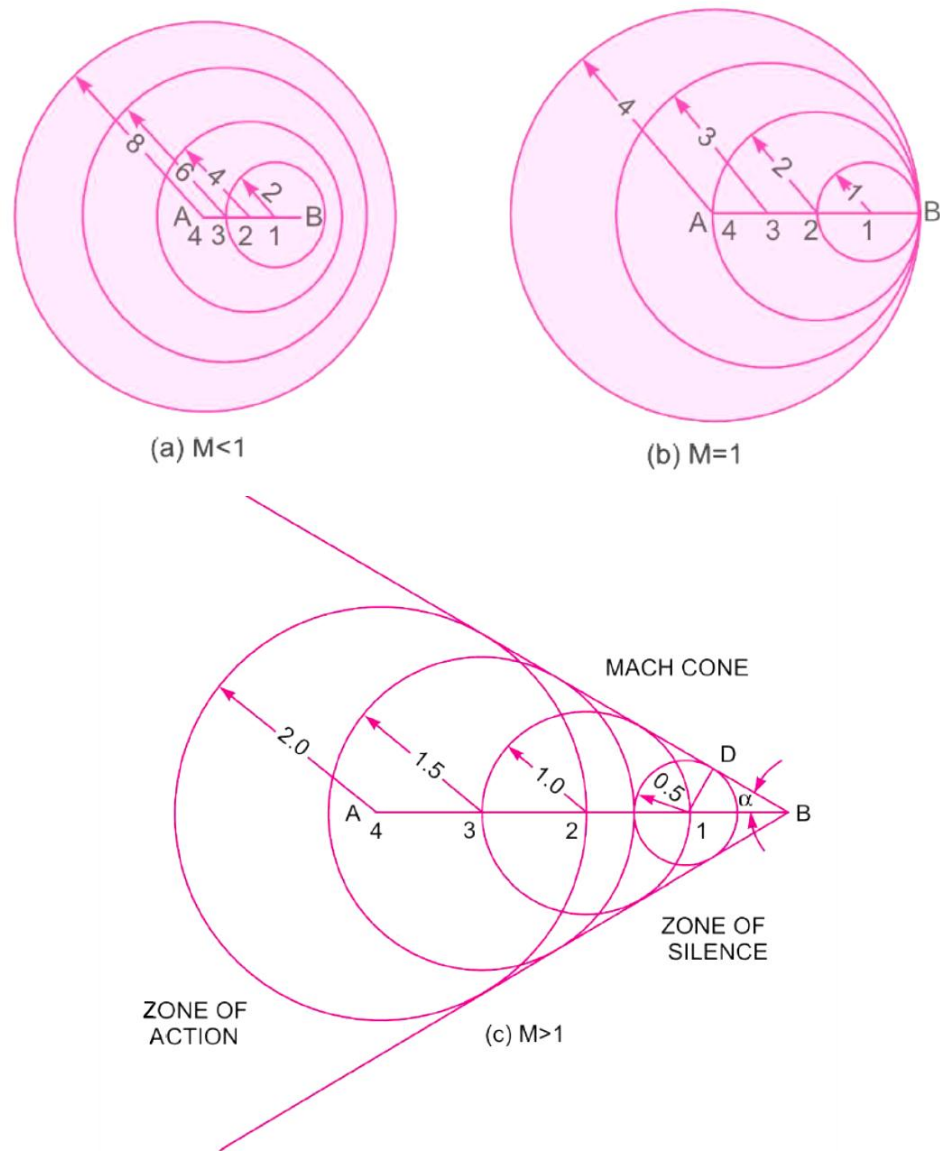


Fig.6.2 – Nature of propagation of disturbances in compressible flow

**Case II. When  $M = 1$  (i.e.,  $V = C$ , Sonic flow) :**

- ▶ In this case since  $V = C$ , let take  $V = 1$  unit and  $C = 1$  units, so that  $\frac{V}{C} = 1$ .
- ▶ Let the projectile moves from A to B in 4 seconds. The disturbance created at A in 4 seconds will move a distance having radius =  $4 \times C = 4 \times 1 = 4$  units in all directions. The projectile from point 3 will move to position B in three seconds and the disturbance created three will move a distance having a radius =  $3 \times C = 3 \times 1 = 3$  units in all directions. Similarly at point 2 and point 1, the disturbance created at these points will move a distance having radius 2 and 1 in all directions respectively as shown in Fig. 6.2 (b).
- ▶ In this case, the disturbance always travels with the projectile as shown in Fig. 6.2 (b).

**Case III. When  $M > 1$  (i.e.,  $V > C$ , Supersonic flow) :**

- ▶ In this case since  $V > C$ , let take  $V = 1$  unit and  $C = 0.5$  units, so that  $\frac{V}{C} = 2$ , which is greater unity.
- ▶ Let the projectile moves from A to B in 4 seconds and hence the distance travel by projectile in 4 seconds  $AB = 4 \times V = 4 \times 1 = 4$  units. The radius of disturbance will equal to  $4 \times C = 4 \times 0.5 = 2$  units in all directions. Hence taking A as a centre and radius equal to 2 units, a circle is drawn. This circle gives position of disturbance after after 4 seconds. After 1 second from A, the projectile is

at point 3, and distance  $A3 = V \times 1 = 1 \times 1 = 1$  units. The projectile from point 3 will reach B in three seconds, the disturbance will have radius  $3 \times C = 3 \times 0.5 = 1.5$  units and similarly the radius of disturbance at point 2 and 1 will be  $2 \times C = 2 \times 0.5 = 1$  unit and  $1 \times C = 1 \times 0.5 = 0.5$  unit respectively as shown in Fig. 6.2 (c).

- ▶ In this case the projectile travels faster than the disturbance. Thus the distance  $AB$  (which the projectile has travelled) is more than ' $Ct$ ', and hence the projectile at point 'B' is outside the spheres formed due to formation and growth of disturbance at  $t = 0$ .

### 6.5.1 Mach Cone

- ▶ If the tangents are drawn (from the point B) to the circles, the spherical pressure waves form a cone with its vertex at B is known as Mach cone.

### 6.5.2 Mach Angle

- ▶ The semi-vertex angle  $\alpha$  of the cone is known as Mach angle.
- ▶ Mach angle is given by,

$$\sin \alpha = \frac{Ct}{Vt} = \frac{C}{V} = \frac{1}{M} \quad \text{Eq. (6.10)}$$

### 6.5.3 Zone Of Action

- ▶ From eq. (6.4), if  $M > 1$ , the effect of the disturbance is felt only in region inside the Mach cone, this region is called zone of action.
- ▶ It has been observed that when an aeroplane is moving with supersonic speed, its noise is heard only after the plane has already passed over us.

### 6.5.4 Zone Of Silence

- ▶ The region outside the Mach cone is called zone of silence.

## 6.6 Static And Stagnation States

- ▶ The point on the immersed body where the velocity is zero is called stagnation point.
- ▶ At this point velocity head is converted into pressure head. The values of pressure ( $p_s$ ), temperature ( $T_s$ ) and density ( $\rho_s$ ), at stagnation point are called stagnation properties.

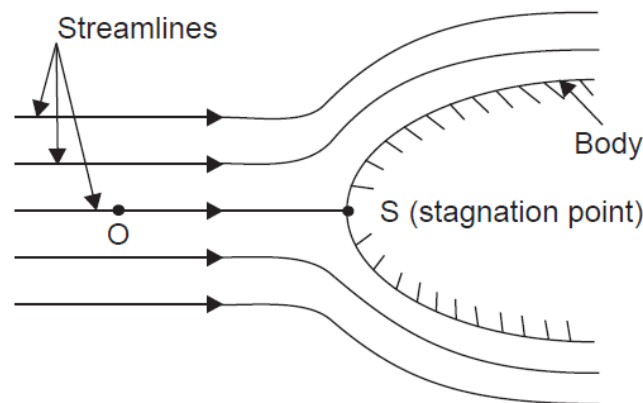


Fig.6.3 – Stagnation properties

- ▶ Consider the flow of compressible fluid past an immersed body where the velocity becomes zero. Consider frictionless adiabatic (isentropic) condition.



- ▶ Let us consider two points, O in the free stream and the stagnation point S as shown in Fig. 6.2.
- ▶ Let,  $p_0$  = pressure of compressible fluid at point O,  
 $V_0$  = velocity of fluid at O,  
 $\rho_0$  = density of fluid at O,  
 $T_0$  = temperature of fluid at O, and

$p_s, V_s, \rho_s,$  and  $T_s$  are corresponding values of pressure, velocity density, and temperature at point 'S'

- ▶ Expression for stagnation pressure ( $p_s$ ) in terms of Mach number is given by,

$$p_s = p_0 \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M_0^2 \right]^{\frac{\gamma}{\gamma - 1}} \quad \text{Eq. (6.11)}$$

- ▶ Expression for stagnation density ( $\rho_s$ ) in terms of Mach number is given by,

$$\rho_s = \rho_0 \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M_0^2 \right]^{\frac{1}{\gamma - 1}} \quad \text{Eq. (6.12)}$$

- ▶ Expression for stagnation temperature ( $T_s$ ) in terms of Mach number is given by,

$$T_s = T_0 \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M_0^2 \right] \quad \text{Eq. (6.13)}$$

## 6.7 Effect Of Mach Number On Compressibility

- ▶ Expression for stagnation pressure ( $p_s$ ) in terms of Mach number is given by,

$$p_s = p_0 \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M_0^2 \right]^{\frac{\gamma}{\gamma - 1}}$$

- ▶ If the right hand side of eqn. (6.11) is expanded by the binomial theorem, we get

$$p_s = p_0 + \frac{p_0 \gamma M_0^2}{2} \left( 1 + \frac{M_0^2}{4} + \frac{2 - \gamma}{24} M_0^4 + \dots \right) \quad \text{Eq. (6.14)}$$

$$\text{But, } M_0^2 = \frac{V_0^2}{C_0^2} = \frac{V_0^2}{\left( \frac{\gamma p_0}{\rho_0} \right)} = \frac{V_0^2 \rho_0}{\gamma p_0} \quad \left( \because C_0^2 = \frac{\gamma p_0}{\rho_0} \right)$$

- ▶ Substituting the value of  $M_0^2$  in eqn. (6.8), we get

$$p_s = p_0 + \frac{p_0 \gamma}{2} \times \frac{V_0^2 \rho_0}{\gamma p_0} \left( 1 + \frac{M_0^2}{4} + \frac{2 - \gamma}{24} M_0^4 + \dots \right)$$

$$p_s = p_0 + \frac{p_0 V_0^2}{2} \left( 1 + \frac{M_0^2}{4} + \frac{2 - \gamma}{24} M_0^4 + \dots \right) \quad \text{Eq. (6.15)}$$

$$\text{Also, } p_s = p_0 + \frac{p_0 V_0^2}{2} \quad (\text{when compressibility effects are neglected}) \quad \text{Eq. (6.16)}$$

- ▶ The comparison of eqns. (6.9) and (6.10) shows that the effects of compressibility are isolated in the bracketed quantity and that these effects depend only upon the Mach number. The bracketed quantity,  $\left( 1 + \frac{M_0^2}{4} + \frac{2 - \gamma}{24} M_0^4 + \dots \right)$  may thus be considered as a compressibility correction factor.
- ▶ It is worth noting that :
  - For  $M < 0.2$ , the compressibility affects the pressure difference ( $p_s - p_0$ ) by less than 1 per cent and the simple formula for flow at constant density is then sufficiently accurate.

- For larger value of  $M$ , as the terms of binomial expansion become significant, the compressibility effect must be taken into account.
- When the Mach number exceeds a value of about 0.3, the Pitot-static tube used for measuring aircraft speed needs calibration to take into account the compressibility effects.

## 6.8 Area Velocity Relationship And Effect Of Variation Of Area For Subsonic, Sonic And Supersonic Flows

- ▶ For an incompressible flow the continuity equation may be expressed as  $AV = \text{constant}$ , which when differentiated gives,

$$AdV + VdA = 0 \text{ OR } \frac{dA}{A} = -\frac{dV}{V} \quad \text{Eq. (6.17)}$$

- ▶ But in case of compressible flow, the continuity equation is given by,  $\rho AV = \text{constant}$ , which can be differentiated to give,

$$\begin{aligned} \rho d(AV) + AVd\rho &= 0 \\ \rho(AdV + VdA) + AVd\rho &= 0 \\ \rho AdV + \rho VdA + AVd\rho &= 0 \end{aligned}$$

- ▶ Dividing both sides by  $\rho AV$ , we get

$$\begin{aligned} \frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} & \quad \text{Eq. (6.18)} \\ \frac{dA}{A} &= -\frac{dV}{V} - \frac{d\rho}{\rho} \end{aligned}$$

- ▶ The Euler's equation for compressible fluid is given by,

$$\frac{dp}{\rho} + VdV + g dz = 0$$

Neglecting the 'z' terms the above equation reduces to,

$$\frac{dp}{\rho} + VdV = 0$$

- ▶ This equation can also be expressed as

$$\begin{aligned} \frac{dp}{\rho} \times \frac{d\rho}{d\rho} + VdV &= 0 \\ \frac{dp}{d\rho} \times \frac{d\rho}{\rho} + VdV &= 0 \quad \left( \because \frac{dp}{d\rho} = C^2 \right) \\ C^2 \times \frac{d\rho}{\rho} + VdV &= 0 \\ C^2 \times \frac{d\rho}{\rho} &= -VdV \\ \frac{d\rho}{\rho} &= -\frac{VdV}{C^2} \end{aligned}$$

- ▶ Substituting the value of  $\frac{d\rho}{\rho}$  in eq. (6.12), we get

$$\frac{dV}{V} + \frac{dA}{A} - \frac{VdV}{C^2} = 0$$

$$\frac{dA}{A} = \frac{VdV}{C^2} - \frac{dV}{V} = \frac{dV}{V} \left( \frac{V^2}{C^2} - 1 \right)$$

$$\frac{dA}{A} = \frac{dV}{V} (M^2 - 1) \left( \because M = \frac{V}{C} \right) \quad \text{Eq. (6.19)}$$

- ▶ Eq. (6.13) give variation of  $\frac{dA}{A}$  for the flow of compressible fluids respectively. The ratios  $\frac{dA}{A}$  and  $\frac{dV}{V}$  are respectively fractional variations in the values of area and flow velocity in the flow passage. From this eq, it is possible to formulate the following conclusions of practical significance.

### Case I. For Subsonic flow ( $M < 1$ )

- ▶ When,  $\frac{dV}{V} > 0$ ;  $\frac{dA}{A} > 0$ ;  $dp < 0$  (convergent nozzle)
- ▶ When,  $\frac{dV}{V} < 0$ ;  $\frac{dA}{A} > 0$ ;  $dp > 0$  (divergent diffuser)

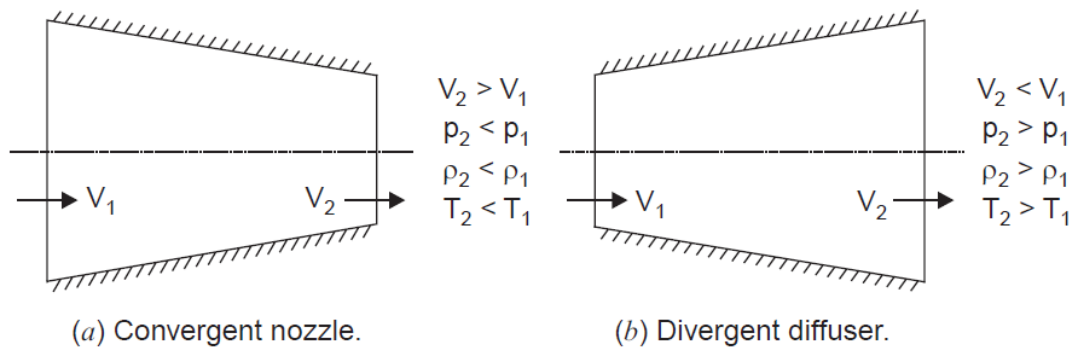


Fig.6.4 – Subsonic flow ( $M < 1$ )

### Case II. For Supersonic flow ( $M > 1$ )

- ▶ When,  $\frac{dV}{V} > 0$ ;  $\frac{dA}{A} > 0$ ;  $dp < 0$  (divergent nozzle)
- ▶ When,  $\frac{dV}{V} < 0$ ;  $\frac{dA}{A} < 0$ ;  $dp > 0$  (convergent diffuser)

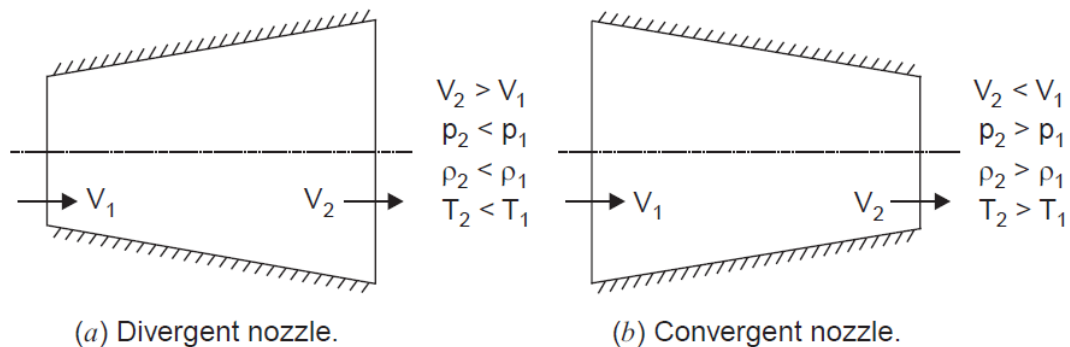


Fig.6.5 – Supersonic flow ( $M > 1$ )

### Case III. For Sonic flow ( $M = 1$ )

- ▶ When  $\frac{dA}{A} = 0$  then straight flow passage since  $dA$  must be zero

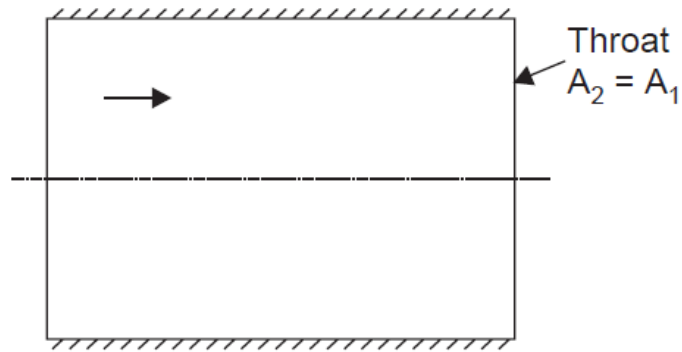


Fig.6.6 – Sonic flow ( $M = 1$ )

- ▶ and  $dp = (\text{zero}/\text{zero})$  i.e., indeterminate, but when evaluated, the change of pressure  $p = 0$ , since  $dA = 0$  and the flow is frictionless.

## 6.9 References

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